Answer Key

Teacher Grading Sheet (PART I)

1. D
2. B
3. B
4. D
5. C
6. D
7. B
8. B
9. D
10. A
11. D
12. D
13. D
14. D
15. C
16. B
17. A
18. A
19. C
20. B
21. B
22. D
23. B
24. B
25. B
26. B
27. C
28. A
29. A
30. B
31. D
32. C
33. D
34. C
35. B
36. A
37. D
38. D
39. C
40. D
41. C
42. B
43. A
44. A
45. C
46. D
47. C
48. C
49. B
50. C

PART II

Total Value 50%

Answer **ALL** items in the space provided. Show **ALL** workings.

Value

4 51. Algebraically determine the EXACT roots, in simplest form, of
$2x^2 + 22 = -8x$.

Marks		
(1)	$2x^2 + 8x + 22 = 0$	$2x^2 + 8x + 22 = 0$
(1)	$x = \frac{-8 \pm \sqrt{(8)^2 - 4(2)(22)}}{2(2)}$	$\frac{2x^2 + 8x + 22}{2} = 0$
	$x = \frac{-8 \pm \sqrt{64 - 176}}{4}$	$x^2 + 4x + 11 = 0$
	$x = \frac{-8 \pm \sqrt{-112}}{4}$	$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(11)}}{2(1)}$
	$x = \frac{-8 \pm \sqrt{16 \times -1 \times 7}}{4}$	$x = \frac{-4 \pm \sqrt{16 - 44}}{2}$
(1)	$x = \frac{-8 \pm 4i\sqrt{7}}{4}$	$x = \frac{-4 \pm \sqrt{-28}}{2}$
(1)	$x = -2 \pm i\sqrt{7}$	$x = \frac{-4 \pm \sqrt{4 \times -1 \times 7}}{2}$
		$x = \frac{-4 \pm 2i\sqrt{7}}{2}$
		$x = -2 \pm i\sqrt{7}$

4 52. Change the equation $y = 2x^2 - 32x + 125$ into transformational form.

Solution A

Marks	
(0.5)	$a = 2$
(1)	$x = -\frac{b}{2a} = -\frac{-32}{2(2)} = 8$
	$\boldsymbol{y = 2x^2 - 32x + 125}$
	$y = 2(8)^2 - 32(8) + 125$
(1)	$y = 128 - 256 + 125$
	$y = -3$
(0.5)	**Therefore the vertex is** $(\mathbf{8}, -\mathbf{3})$
(1)	**Equation is** $\frac{1}{2}(y + 3) = (x - 8)^2$

Solution B

Marks	
	$y = 2x^2 - 32x + 125$
(1)	$y = 2(x^2 - 16x) + 125$
(1)	$y = 2(x^2 - 16x + 64) + 125 - 128$
(1)	$y = 2(x - 8)^2 - 3$
(1)	$\frac{1}{2}(y + 3) = (x - 8)^2$

Value

4 **53.** Clayton has a square shed. In order to accommodate a new snowmobile, the length of the floor must be increased by 2m and the width increased by 1m. If the new floor area is $42m^2$, algebraically determine the original dimensions of the floor.

Let: $x + 1 =$ new width

$x + 2 =$ new length

Marks

$$A = l \times w$$

(1) $42 = (x+1)(x+2)$

$42 = 2 + x + 2x + x^2$

$42 = x^2 + 3x + 2$

$0 = x^2 + 3x + (-40)$

(1) $0 = (x+8)(x-5)$

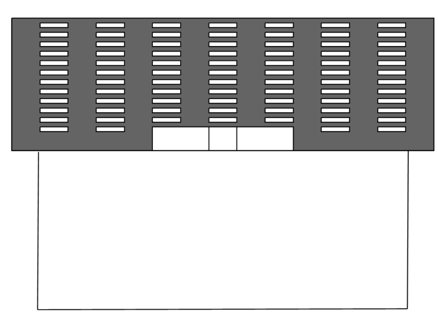
$(x+8) = 0 \qquad (x-5) = 0$

(1.5) $x = -8 \qquad x = 5$

(0.5) Reject $x = -8$

The original dimensions of the shed are $5m \times 5m$.

4 **54.** The school administration has 80 metres of fencing to construct a rectangular playground. If they use a wall of the school as a fourth side, what dimensions will yield a maximum area?



Marks

Let $x = width$ and $y = length$

1 $2x + y = 80$

$y = 80 - 2x$

$A = xy$

1 $A = x(80 - 2x)$

$A = 80x - 2x^2$

$A = -2x^2 + 80x$

$a = -2 \text{ and } b = 80$

1 $x = -\frac{b}{2a} = -\frac{80}{2(-2)} = 20m$

1 $y = 80 - 2x = 80 - 2(20) = 40m$

Value

4 55. A person throws a snowball and the snowball follows a parabolic path that is described by the function $h(t) = -\frac{1}{2}t^2 + 4t + 2$ where t is time in seconds and $h(t)$ is height in metres. Algebraically, determine the maximum height of the snowball and the time at which the maximum height occurs.

Solution A

Marks

(2) $\boldsymbol{t = \frac{-b}{2a} = \frac{-4}{2(-\frac{1}{2})} = 4\ s}$

$h(t) = -\frac{1}{2}t^2 + 4t + 2$

$\boldsymbol{h(4) = -\frac{1}{2}(4)^2 + 4(4) + 2}$

(2) $\boldsymbol{h(4) = 10m}$

So the maximum height of the snowball is 10 metres at 4 seconds.

Solution B

Marks

$\boldsymbol{h(t) = -\frac{1}{2}t^2 + 4t + 2}$

(1) $\boldsymbol{h(t) = -\frac{1}{2}(t^2 - 8t) + 2}$

(1) $\boldsymbol{h(t) = -\frac{1}{2}(t^2 - 8t + 16) + 4}$

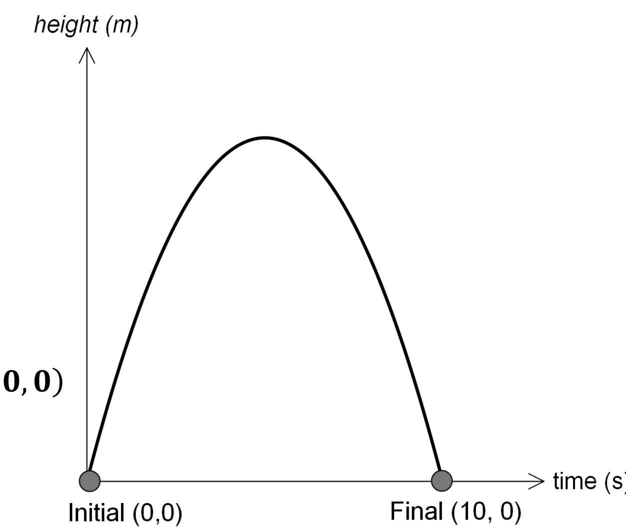
$\boldsymbol{h(t) = -\frac{1}{2}(t - 4)^2 + 20}$

(1) $\boldsymbol{-2(h(t) - 20) = (t - 4)^2}$

Therefore, the vertex is (4, 20)

(1) $\boldsymbol{t = 4s}$ **and** $\boldsymbol{h = 20m}$

4 56. A golfer hits a ball and it reaches a maximum height of 30m. The ball hits the ground in 10 seconds. Algebraically determine the quadratic function that models the situation.



Mark

(1) $(h,k) = (5,30)$

$(y-k) = a(x-h)^2$

$(y-30) = a(x-5)^2$

(1) $(0-30) = a(10-5)^2$; $(x,y) = (10,0)$

$(-30) = a(5)^2$

(1) $-\frac{6}{5} = a$

(1) $-\frac{5}{6}(y-30) = (x-5)^2$

Value

4 57. A blue jay swoops down from the top of a 10m tree to chase away some house sparrows. The blue jay's path follows a parabolic path given by the function $h(t) = 2t^2 - 8t + 10$ where t is time in seconds and $h(t)$ is height in metres. Algebraically, determine the time(s) when the blue jay reaches a height of 4m.

Marks:

$h(t) = 2t^2 - 8t + 10$

(1) $4 = 2t^2 - 8t + 10$

$0 = 2t^2 - 8t + 6$

(1) $0 = t^2 - 4t + 3$

(1) $0 = (t-3)(t-1)$

(1) $t = 3, t = 1$

The blue jay reaches a height of 4 meters at 1 second and at 3 seconds.

4 58. Jim enjoys snowboarding at Marble Mountain. He attempts a jump and his height $h(t)$, in metres, is recorded as $h(t) = -5t^2 + 12t + 15$. Calculate Jim's approximate instantaneous rate of change at $t = 3$ seconds.

	Substitute $\mathbf{t} = \mathbf{2.99}$
Mark	
(1)	$\boldsymbol{h(2.99) = -5t^2 + 12t + 15 = -5(2.99)^2 + 12(2.99) + 15 = 6.1795}$
	Substitute $\mathbf{t} = \mathbf{3.01}$
(1)	$\boldsymbol{h(3.01) = -5t^2 + 12t + 15 = -5(3.01)^2 + 12(3.01) + 15 = 5.8195}$
	Instantaneous Rate of Change (I.R.O.C)
(0.5)	$\text{IROC} = \dfrac{\boldsymbol{h(3.01) - h(2.99)}}{\mathbf{3.01 - 2.99}}$
(0.5)	$\boldsymbol{IROC = \frac{5.8195 - 6.1795}{3.01 - 2.99}}$
	$\boldsymbol{IROC = \frac{-0.36}{0.02}}$
(1.0)	$\boldsymbol{IROC = -18\ m/s}$

Note: The interval could be $\boldsymbol{x \pm 0.1}$

4 59. A sugar cube is placed in a cup of hot coffee. As it dissolves, its volume is given by $V = (15 - 2t)^3$, where t is time in minutes and V is volume in mm^3. Calculate the average rate of change in the volume between 5 and 6 minutes.

Substitute t = 5	
Mark	
(1)	$\boldsymbol{h(5) = (15 - 2t)^3 = (15 - 2(5))^3 = 125}$
Substitute t = 6	
Mark	
(1)	$\boldsymbol{h(6) = (15 - 2t)^3 = (15 - 2(6))^3 = 27}$
Average Rate of Change (A.R.O.C)	
Marks:	
(0.5)	AROC $= \frac{h(6) - h(5)}{6-5}$
(0.5)	$AROC = \frac{27-125}{6-5}$
	$AROC = \frac{-98}{1}$
(1.0)	$AROC = -98\ mm^3/min$

4 60. Sean likes to spend money. He currently has \$1000 in his bank account but unfortunately, due to his spending habits, his account has a half-life of 3 years.

Andrew likes to save money. He invests \$100 in a fund that doubles every three years.

Write the exponential functions that model their situations and use them to determine who will have more money in 5 years.

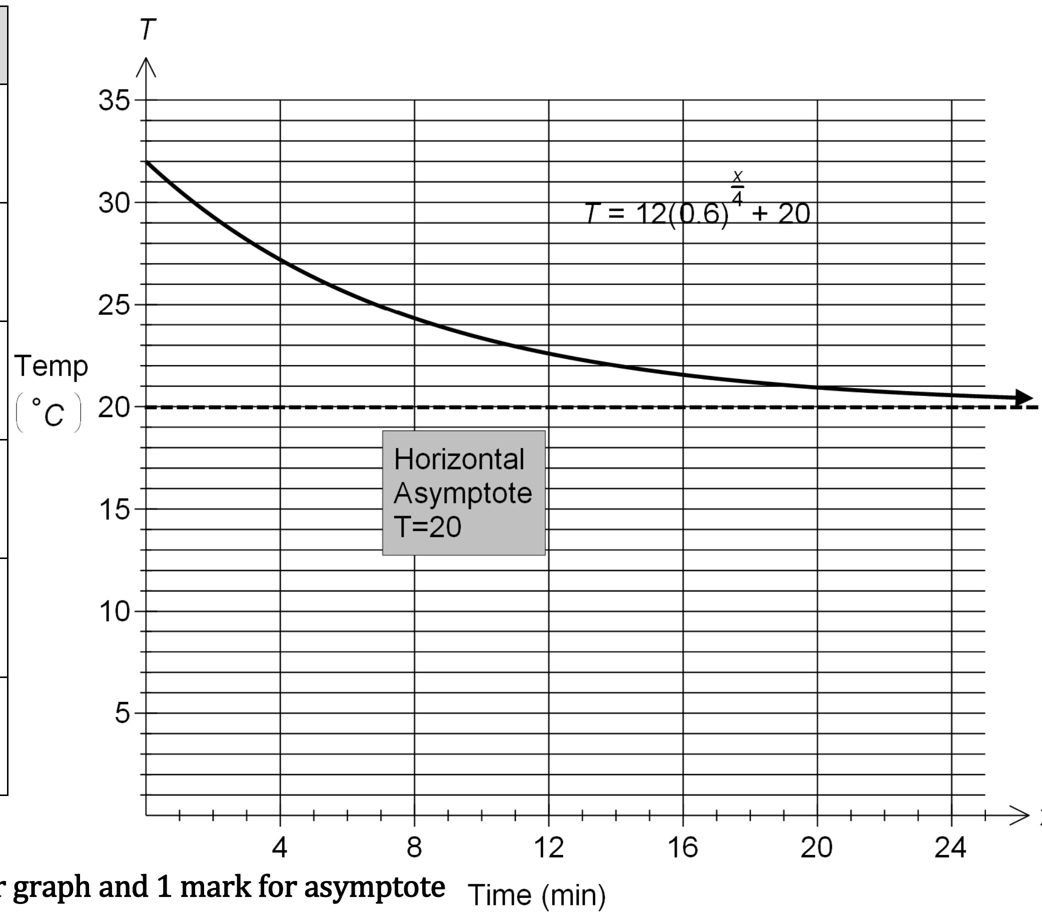
Sean		Andrew	
Mark		Mark	
	$y = A_o\left(\frac{1}{2}\right)^{\frac{x}{d}}$		$y = A_o(2)^{\frac{x}{d}}$
(0.5)	$y = 1000\left(\frac{1}{2}\right)^{\frac{x}{3}}$	(0.5)	$y = 100(2)^{\frac{x}{3}}$
(0.5)	$y = 1000\left(\frac{1}{2}\right)^{\frac{5}{3}}$	(0.5)	$y = 100(2)^{\frac{5}{3}}$
(0.5)	$y = \$314.98$	(0.5)	$y = 317.48$

(1.0) Andrew will have more money than Sean.

Value

3 61. Your computer registers the temperature of its motherboard as 32°C. During a power outage, the cooling of the motherboard is modeled by $T = 12(0.6)^{\frac{x}{4}} + 20$, where T is the temperature in degrees Celsius and x is time in minutes. Complete the table of values for the first 20 minutes, then sketch the graph including the asymptote.

x in minutes	T in Celsius
0	**32**
4	**27.2**
8	**24.3**
12	**22.6**
16	**21.6**
20	**20.9**



1 mark for table, 1 mark for graph and 1 mark for asymptote

3 62. Karen purchased a house in 1995. In 2012, it is worth \$285 000. If the house has appreciated by 11% every 2 years, what was the original cost of the house?

Mark

$A = A_o b^{\frac{t}{c}}$ *where* $A = 285\,000, b = 1 + 0.11 = 1.11, c = 2, t = 17$

(1) $A = A_o(1.11)^{\frac{t}{2}}$

(1) $285\,000 = A_o(1.11)^{\frac{17}{2}}$

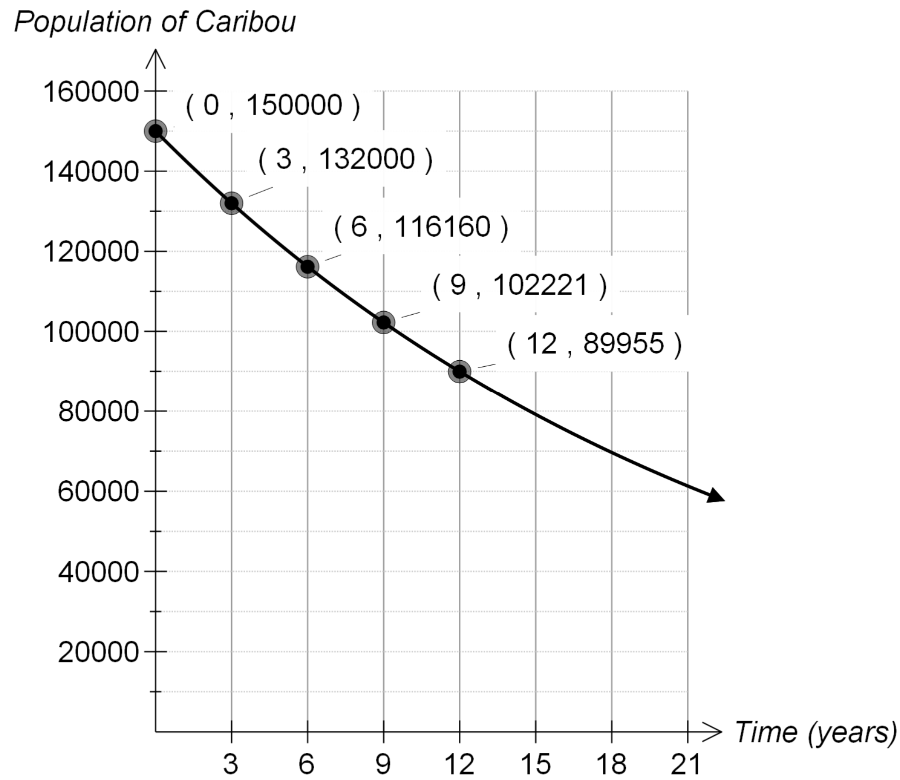
(0.5) $285\,000 = A_o(2.4279812)$

(0.5) $A_0 = \$117\,381.47$

The original cost of the house was \$117 281.47.

Value

4 63. The graph shows the decrease in the population of Woodland Caribou in Newfoundland over the past 12 years. Algebraically, determine an equation that models the population of caribou, P, over time, t. Use the equation to predict the caribou population in year 25.



Marks

$\boldsymbol{P_o = 150\ 000,}$

(1) $\boldsymbol{b = 0.88}$

$\boldsymbol{c = 3}$

(1) $\boldsymbol{P = 150\ 000(0.88)^{\frac{t}{3}}}$

(1) $\boldsymbol{P = 150\ 000(0.88)^{\frac{25}{3}}}$

(1) $\boldsymbol{P = 51\ 695}$