Teacher Grading Sheet (PART I)

1. C
2. A
3. D
4. D
5. A
6. A
7. B
8. B
9. A
10. B
11. D
12. D
13. C
14. A
15. B
16. C
17. C
18. C
19. B
20. B
21. A
22. D
23. A
24. C
25. D
26. A
27. A
28. C
29. A
30. D
31. A
32. C
33. D
34. C
35. B
36. C
37. D
38. D
39. B
40. A
41. C
42. A
43. A
44. A
45. B
46. C
47. D
48. C
49. B
50. C

PART II

Total Value 50%

Answer **ALL** items in the space provided. Show **ALL** workings.

Value

4 51. Algebraically determine the EXACT roots, in simplest form, of $5x^2 + 3 = 2x(x + 4)$.

Marks

(1) $3x^2 - 8x + 3 = 0$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(3)}}{2(3)}$

(1) $x = \frac{8 \pm \sqrt{64 - 36}}{6}$

$x = \frac{8 \pm \sqrt{28}}{6}$

(1) $x = \frac{8 \pm 2\sqrt{7}}{6}$

(1) $x = \frac{4 \pm \sqrt{7}}{3}$

4 52. Change the equation $y = 5x^2 + 40x + 83$ into transformational form.

Solution A

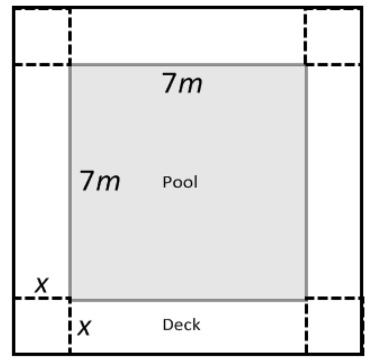
Marks	
(0.5)	$a = 5$
(1)	$x = \frac{-b}{2a} = \frac{-40}{2(5)} = \frac{-40}{10} = -4$
	$y = 5x^2 + 40x + 83$
	$y = 5(-4)^2 + 40(-4) + 83$
(1)	$y = 80 - 160 + 83 = -80 + 83 = 3$
(0.5)	Therefore the vertex is $(-4, 3)$
(1)	Equation is $\frac{1}{5}(y - 3) = (x + 4)^2$

Solution B

Marks	
	$y = 5x^2 + 40x + 83$
(0.5)	$y - 83 = 5x^2 + 40x$
(1)	$y - 83 = 5(x^2 + 8x)$
(1)	$y - 83 + 80 = 5(x^2 + 8x + 16)$
(0.5)	$(y - 3) = 5(x + 4)^2$
(1)	$\frac{1}{5}(y - 3) = (x + 4)^2$

Value

4 53. A swimming pool measuring 7 m by 7 m is surrounded by a deck of uniform width. If the combined area of the pool and the deck is 121 m^2, algebraically find the width of the deck.



Marks

$$A = l \times w$$

(1) $121 = (7 + 2x)(7 + 2x)$

$121 = 49 + 14x + 14x + 4x^2$

$0 = 4x^2 + 28x - 72$

$0 = 4(x^2 + 7x - 18)$

$0 = 4(x + 9)(x - 2)$

$(x + 9) = 0$ $\quad$ $(x - 2) = 0$

(1.5) $x = -9$ $\quad$ $x = 2$

(0.5) Reject $x = -9$

(1) So the width of the deck is 2 metres

4 54. Algebraically determine two **positive** consecutive numbers whose product is 210.

Marks

(0.5) Let x be the smallest number and $x + 1$ be the largest number

(0.5) $P = x(x + 1)$

$210 = x^2 + x$

(1) $0 = x^2 + x - 210$

$0 = (x + 15)(x - 14)$

$(x + 15) = 0$ $\quad$ $(x - 14) = 0$

(1) $x = -15$ $\quad$ $x = 14$

(0.5) Reject $x = -15$

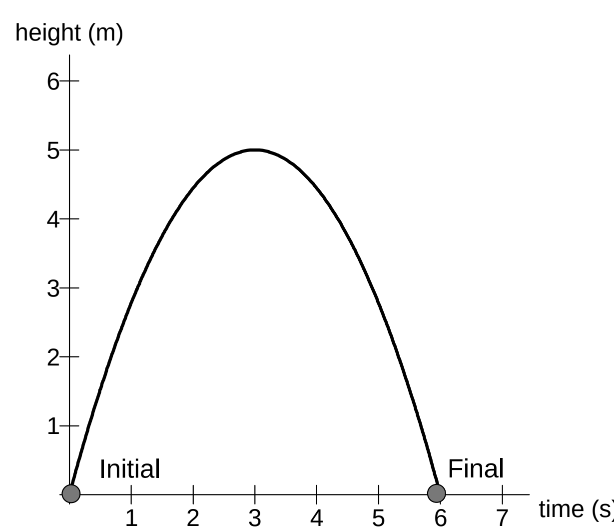
(0.5) Therefore, the two positive consecutive numbers are $x = 14$ and $x + 1 = 15$.

Value

4 55. A student throws his calculator and the calculator follows a parabolic path that is described by the function $h(t) = -2t^2 + 8t + 13$ where t is the time in seconds and $h(t)$ is the height in metres. **Algebraically** determine the maximum height of the calculator and the time at which the maximum height occurs?

Solution A	
Marks	
(1.5)	$t = \frac{-b}{2a} = \frac{-8}{2(-2)} = 2$
	$h(t) = -2t^2 + 8t + 13$
	$h(t) = -2(2)^2 + 8(2) + 13$
(1.5)	$h(t) = 21$
(1)	**So the maximum height of the calculator is 21 metres at 2 seconds.**
Solution B	
Marks	$h(t) = -2t^2 + 8t + 13$
(1)	$h(t) = -2(t^2 - 4t) + 13$
(1)	$h(t) = -2(t^2 - 4t + 4) + 13 + 8$
	$h(t) = -2(t-2)^2 + 21$
(1)	$-\frac{1}{2}(h(t) - 21) = (t-2)^2$
	Therefore, the vertex is (2, 21)
(1)	$t = 2s$ **and** $h = 21m$

4 56. A child kicks a soccer ball and it reaches a maximum height of 5 m in 3 s. The soccer ball hits the ground in 6 s. Algebraically determine the quadratic function that models the situation.



Mark

(1) $(x, y) = (6, 0), (h, k) = (3, 5)$

$\frac{1}{a}(y - k) = (x - h)^2$

$\frac{1}{a}(y - 5) = (x - 3)^2$

(1) $\frac{1}{a}(0 - 5) = (6 - 3)^2$

$\frac{1}{a}(-5) = (3)^2$

(1) $\frac{1}{a} = -\frac{9}{5}$

(1) $-\frac{9}{5}(y - 5) = (x - 3)^2$

Value

4 57. A crow at the top of a 22 m building swoops down to chase away a blue jay. The crow follows a parabolic path given by the function $h(t) = 2t^2 - 12t + 22$, where $h(t)$ is the height in metres and t is the time in seconds. Algebraically, determine the time(s) when the crow reaches a height of 12 m.

Marks:	
	$\boldsymbol{h(t) = 2t^2 - 12t + 22}$
(0.5)	$\boldsymbol{12 = 2t^2 - 12t + 22}$
	$\boldsymbol{0 = 2t^2 - 12t + 10}$
(1)	$\boldsymbol{0 = t^2 - 6t + 5}$
(1.5)	$\boldsymbol{0 = (t-5)(t-1)}$
(1)	$\boldsymbol{t = 5}$, **t=1**

The crow reaches a height of 3 metres at 1 second and at 5 seconds.

4 58. The path of a toy rocket is modeled by $h(t) = -2t^2 + 50t$ where $h(t)$ is the height in metres and t is the time in seconds. What is the average rate of change in the height of the toy rocket between $t = 1$ and $t = 3$?

Substitute t = 1	
Mark	
(1)	$\boldsymbol{h(1) = -2t^2 + 50t = -2(1)^2 + 50(1) = -2 + 50 = 48}$
Substitute t = 3	
Mark	
(1)	$\boldsymbol{h(3) = -2t^2 + 50t = -2(3)^2 + 50(3) = -18 + 150 = 132}$
Average Rate of Change (A.R.O.C)	
Marks:	
(0.5)	AROC $= \dfrac{\boldsymbol{h(3) - h(1)}}{\boldsymbol{3-1}}$
(0.5)	$\boldsymbol{AROC = \frac{132-48}{3-1}}$
	$\boldsymbol{AROC = \frac{84}{2}}$
(1.0)	$\boldsymbol{AROC = 42\ m/s}$

Value

4 59. The path of a moving particle is given by the equation $s = 11t^2 - 7t - 10$, where s is the distance in metres and t is the time in seconds. Algebraically determine the instantaneous rate of change in the distance at 3 seconds?

Substitute $t = 2.9$ (or other appropriate values)	
Mark	
(1)	$s(2.9) = 112.92-72.9-10=92.51-20.3-10=62.21$
Substitute $t = 3.1$ (or other appropriate values)	
Mark	
(1)	$s(3.1) = 11(3.1)^2 - 7(3.1) - 10 = 105.71 - 21.7 - 10 = 74.01$
Instantaneous Rate of Change (I.R.O.C)	
Marks:	
(0.5)	$\text{IROC} = \frac{s(3.1)-s(2.9)}{3.1-2.9}$
(0.5)	$IROC = \frac{74.01-62.21}{3.1-2.9}$
	$IROC = \frac{11.8}{0.2}$
(1.0)	$IROC = 59\ m/s$

3 60. An element has a half-life of 150 years. If its initial mass is 60 grams, determine an exponential function and use it to find the amount of the element remaining after 450 years?

Mark

$A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{d}}$

(1) $A(t) = 60\left(\frac{1}{2}\right)^{\frac{t}{150}}$

(1) $A(450) = 60\left(\frac{1}{2}\right)^{\frac{450}{150}}$

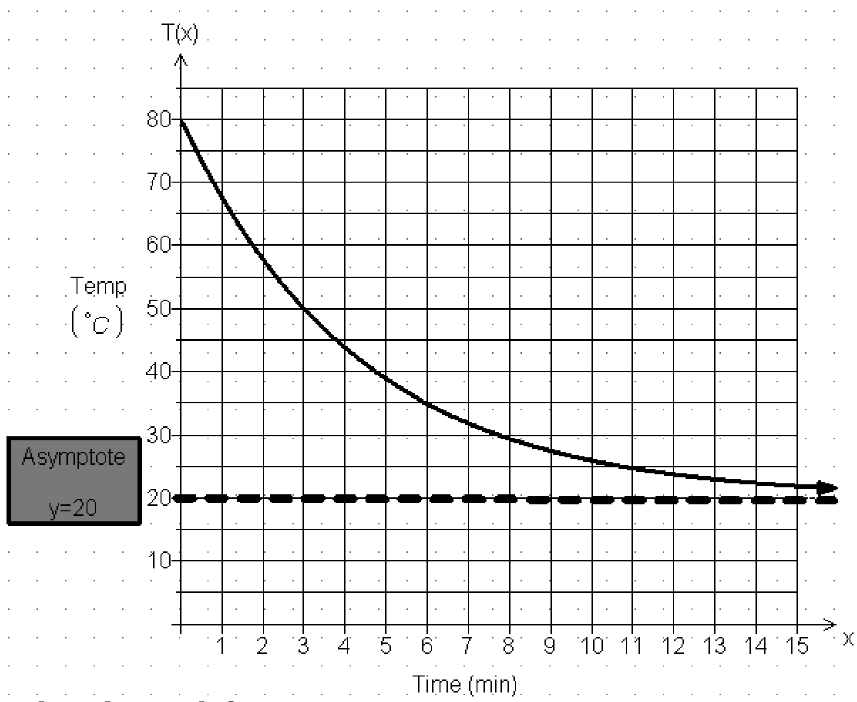
$A(450) = 60\left(\frac{1}{2}\right)^{3}$

(1) $A(450) = 7.5\ grams$

Value

3 61. $T(x) = 60\left(\frac{1}{2}\right)^{\frac{x}{3}} + 20$ represents the temperature of a cooling liquid over time, where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes. Complete the table of values provided and sketch the graph of the function, including the asymptote.

x in minutes	$T(x)$ in Celsius
0	**80**
3	**50**
6	**35**
9	**27.5**
12	**23.75**



1 mark for table, 1 mark for graph and 1 mark for asymptote

4 62. A skateboard was purchased in 2006 and depreciated 8% every 2 years. If the current value of the skateboard in 2011 is \$180, determine an exponential function and use it to find the original value of the skateboard.

Mark

(1) $\boldsymbol{A = A_o b^{\frac{t}{c}}}$ *where* $A = 180, b = 1 - 0.08 = 0.92, c = 2, t = 5$

(1) $\boldsymbol{A = A_o(0.92)^{\frac{t}{2}}}$

(1) $\boldsymbol{180 = A_o(0.92)^{\frac{5}{2}}}$

(0.5) $\boldsymbol{180 = A_o(0.81184)}$

(0.5) $\boldsymbol{A_0 = 221.71}$

The original cost of the skateboard was \$221.71.

Value

4 63. Initially there were 50 cell phones in a community. The number of cell phones increased according to the data in the table. Algebraically determine an equation that models the number of cell phones, C, over time, t. Use the equation to predict the number of cell phones after 25 years.

Time in years, t	0	4	8	12	16
Number of cell phones, C	50	100	200	400	800

Mark

(1) $\boldsymbol{C_o = 50, b = 2, c = 4}$

(1) C= $\mathbf{50(2)^{\frac{x}{4}}}$

$\boldsymbol{C = 50(2)^{\frac{25}{4}}}$

$\boldsymbol{C = 50(2)^{6.25}}$

(1) $\boldsymbol{C = 50(76.10925536)}$

(1) $\boldsymbol{C = 3805.4628}$

After 25 years there will be about 3805 cell phones in the community.