Teacher Grading Sheet (PART I)

1. A
2. B
3. B
4. C
5. C
6. B
7. D
8. A
9. D
10. B
11. D
12. A
13. C
14. A
15. D
16. B
17. D
18. A
19. C
20. B
21. D
22. D
23. D
24. A
25. D
26. C
27. B
28. A
29. D
30. D
31. A
32. D
33. D
34. A
35. C
36. D
37. A
38. D
39. A
40. A
41. A
42. D
43. A
44. A
45. B
46. C
47. A
48. C
49. B
50. C

Student’s Name: ______________

Teacher’s Name: ______________

PART II

Total Value 50%

Answer **ALL** items in the space provided. Show **ALL** workings.

Value

4 51. Algebraically determine the **EXACT** roots, in simplest form, of $x(x-3)=-9$.

Marks

1	$x^2-3x+9=0$
1	$x=\frac{3\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}$
	$x=\frac{3\pm\sqrt{9-36}}{2}$
1	$x=\frac{3\pm\sqrt{-27}}{2}$
1	$x=\frac{3\pm 3i\sqrt{3}}{2}$

4 52. Change the equation $y=4x^2+16x+70$ into transformational form.

Either

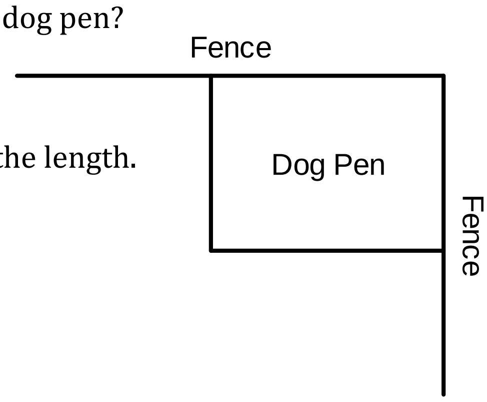
0.5	$y=4(x^2+4x)+70$
1	$y=4(x^2+4x+4-4)+70$
1	$y=4(x+2)^2-16+70$
0.5	$y=4(x+2)^2+54$
1	$\frac{1}{4}(y-54)=(x+2)^2$

Or

0.5	$a = 4$
1	$h=-\frac{b}{2a}=-\frac{16}{2(4)}=-\frac{16}{8}=-2$
	$k=4(-2)^2+16(-2)+70$
	$k=4(4)+(-32)+70$
1	$k=16+38$=54
0.5	Vertex is (-2, 54)
1	Equation is $\frac{1}{4}(y-54)=(x+2)^2$

Value

4 53. You want to construct a dog pen, which has an area of $60m^2$, in the corner of your fenced-in backyard. If the length is $7m$ more than the width, **algebraically** determine the dimensions of the dog pen?



Marks

Let **x** be the width and **x + 7** be the length.

$A = x(x + 7) = 60$

1 $x(x + 7) = 60$

$x^2 + 7x - 60 = 0$

$(x + 12)(x - 5) = 0$

1.5 So $x = -12$ or $x = 5$

0.5 Reject $x = -12$

So the width is $x = 5$ and the length is $x = 5 + 7 = 12$

1 Thus the dimensions are 5m and 12m.

4 54. The sum of two numbers is 10. The sum of their squares is a minimum. **Algebraically** determine the two numbers.

Let x equal one number and y equal the other number.

1 x= first number, so x-10 = second number

0.5 $S = x^2 + (x - 10)^2$

$S = x^2 + x^2 - 20x + 100$

1 $S = 2x^2 - 20x + 100$

Either	**Or**
$x = -\frac{b}{2a} = \frac{20}{2(2)} = 5$	$S = 2(x^2 - 10x) + 100$
1 $x = 5$	$S = 2(x^2 - 10x + 25) + 100 - 50$
	$S = 2(x - 5)^2 + 50$
	$\frac{1}{2}(S - 50) = (x - 5)^2$
	$x = 5$

The first number is 5.

The *second number* $= x - 10 = 5 - 10 = -5$

0.5 So the second number is -5.

Therefore the 2 numbers are 5 and -5.

Value

4 **55.** A volleyball player is serving the ball and the ball follows a parabolic path that is described by the function $h(t) = -8t^2 + 8t + 1$, where t is the time in seconds and h is the height in metres. **Algebraically** determine the maximum height of the volleyball and the time at which the maximum height occurs?

Marks

Either

1.5 $t = \frac{-b}{2a} = \frac{-8}{2(-8)} = \frac{-8}{-16} = \frac{1}{2}$

$h = -8(0.5)^2 + 8(0.5) + 1$

$h = -2 + 4 + 1$

1.5 $h = 3$

1 So the maximum height of 3 m was reached at 0.5 seconds.

Or

1 $h = -8(t^2 - 1t) + 1$

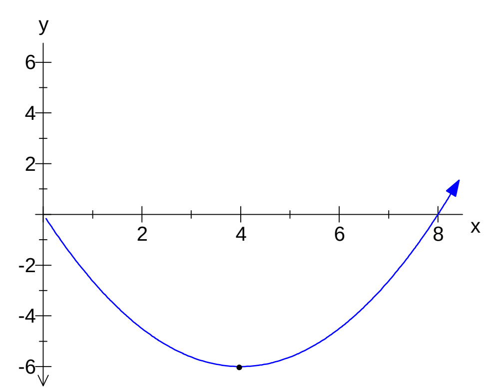
1 $h = -8(t^2 - 1t + 0.25) + 1 + 2$

1 $-\frac{1}{8}(h - 3) = (t - 0.5)^2$

Vertex is (0.5, 3), so

1 $t = 0.5$ s and $h = 3$ m

4 **56.** A seal dives beneath the surface of the ocean to catch a codfish 6 m below. **Algebraically** determine the quadratic function if the seal resurfaces 8 m from his initial position.



Marks

1 $(x, y) = (0,0), (h, k) = (4, -6)$

$\frac{1}{a}(y - k) = (x - h)^2$

$\frac{1}{a}(y + 6) = (x - 4)^2$

1 $\frac{1}{a}\left(0 - (-6)\right) = (0 - 4)^2$

$\frac{1}{a}(6) = 16$

1 $\frac{1}{a} = \frac{16}{6} = \frac{8}{3}$

$\frac{8}{3}\left(y - (-6)\right) = (x - 4)^2$

1 $\frac{8}{3}(y + 6) = (x - 4)^2$

Value

4 57. A cannonball is launched into the air. It follows a parabolic path described by the function $h(t) = -2t^2 + 20t + 3$, where t is the time in seconds and $h(t)$ is the height in metres. **Algebraically** determine when the cannonball will reach a height of 45 m?

Marks

0.5	$45 = -2t^2 + 20t + 3$
	$-2t^2 + 20t - 42 = 0$
1	$t^2 - 10t + 21 = 0$
1.5	$(t-7)(t-3) = 0$
1	$t = 7, t = 3$

4 58. The amount of medicine in your blood stream is modeled by $A = 40(0.8)^t$ where A is the amount of medicine in mg and t is the time in hours. Algebraically determine the average rate of change in the amount of medicine in your bloodstream between 3 hours and 5 hours.

0.5	$AROC = \frac{A(5)-A(3)}{5-3}$
1	$A(3) = 40(0.8)^t = 40(0.8)^3 = 20.48$
1	$A(5) = 40(0.8)^t = 40(0.8)^5 = 13.1072$
0.5	$\text{AROC} = \frac{13.1072-20.48}{2}$
	$\text{AROC} = \frac{-7.372}{2}$
1	$\text{AROC} = -3.68\ mg/hr$ or $-3.7\ mg/hr$

4 59. The height, h (in metres), of a ball is given by the equation $h = -4.9t^2 + 30t$ where t is the time in seconds after the ball is hit. Algebraically determine the instantaneous rate of change in the ball's height above the ground at 2.0 s.

Use t=2.1 s and t=1.9 s (or other appropriate values)

0.5	$IROC = \frac{h(2.1)-h(1.9)}{2.1-1.9}$
1	$h(1.9) = -4.9(1.9)^2 + 30(1.9) = 39.311$
1	$h(2.1) = -4.9(2.1)^2 + 30(2.1) = 41.391$
0.5	$IROC = \frac{41.391-39.311}{0.2}$
	$IROC = \frac{2.08}{0.2}$
1	$IROC = 10.4\ m/s$

3 60. A person invests \$500. It doubles in value every 6 years. Algebraically determine how much his investment is worth in 42 years.

Marks

$$A(t) = A_0(2)^{\frac{t}{d}}$$

1 $$A(t) = 500(2)^{\frac{t}{6}}$$

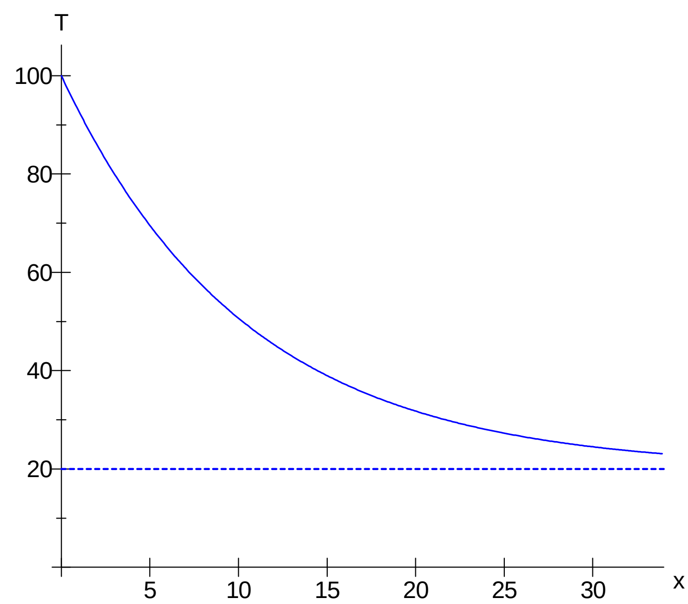
1 $$A(42) = 500(2)^{\frac{42}{6}}$$

$$A(42) = 500(2)^7$$

1 $$= \$64\ 000$$

3 61. A Delissio Pizza is taken out of the oven and laid on the kitchen table to cool. Its temperature is modeled by $T = 80(0.75)^{\frac{x}{3}} + 20$, where T is the temperature in degrees Celsius and x is the time in minutes. Complete the table of values for the first 30 minutes and sketch the graph, including the asymptote.

x	T
0	100
5	69.5
10	50.7
15	39
20	31.8
25	27.3
30	24.5



1 mark for table, 1 mark for graph, 1 mark for asymptote

4 62. A snowmobile was purchased in 1998 and depreciated by 12% every 3 years. If the current value of the snowmobile is \$7196, what was the original value of the snowmobile?

1 $A = A_0 b^{\frac{t}{c}}$ where $A = 7196, b = 1 - 0.12 = 0.88, c = 3, t = 12$

1 $7196 = A_0(0.88)^{\frac{12}{3}}$

1 $7196 = A_0(0.88)^4$

0.5 $7196 = 0.599695A_0$

0.5 $11999.43 = A_0$

Value

4 63. The table below shows the number of salmon counted on Harry's River over a period of time. Algebraically determine an equation that models the number of salmon, *n*, over time, *t*. Use the equation to predict the salmon population after 32 years.

Time (t) in years	0	4	8	12	16
Number of salmon, n	10 000	5000	2500	1250	625

Marks

1 $N_0 = 10000, b = 0.5, c = 4$

1 $N = 10000(0.5)^{\frac{x}{4}}$

$N = 10000(0.5)^{\frac{32}{4}}$

$N = 10000(0.5)^8$

1 $N = 10000(0.00390625)$

$N = 39.0625$

1 After 32 years there will be 39 salmon.