



STUDENT'S NAME \_\_\_\_\_

TEACHER'S NAME \_\_\_\_\_

**DO NOT OPEN THIS EXAMINATION PAPER UNTIL  
YOU ARE TOLD BY THE SUPERVISOR TO BEGIN**

# MIDYEAR EXAMINATION MATHEMATICS 3204

January 27, 2009

Value 100 Marks

Time: 3 hours

## ***General Instructions***

1. Students are required to do **ALL** items.
2. The examination consists of the following parts:  

|                 |                      |            |
|-----------------|----------------------|------------|
| <b>PART I:</b>  | Selected Response    | Value: 50% |
| <b>PART II:</b> | Constructed Response | Value: 50% |
3. A self-powered calculator may be used for calculations and to obtain special values.
4. Answers to **PART I** items are to be shaded on the computer scorable answer sheet. If a second sheet is provided for **PART I** items, letters should be clearly written and this sheet ***stapled to the front*** of the examination paper.
5. For **PART II** items, students are reminded to show all necessary steps and calculations as credit may be given for incomplete or for partially correct solutions. Correct answers without calculations will not merit full marks.

## ***Student Checklist***

***The following items are your responsibility. Please ensure that they are completed.***

- Check that you are doing the correct exam.
- Write your name on the top of this page and on any answer sheet.
- Check that the bubble sheet is adequately shaded.
- Check this exam to ensure that there are no missing pages.
- At the end of the examination period check that you have completed or at least attempted **ALL** items.

## Teacher Grading Sheet (PART I)

- |           |           |
|-----------|-----------|
| 1. _____  | 26. _____ |
| 2. _____  | 27. _____ |
| 3. _____  | 28. _____ |
| 4. _____  | 29. _____ |
| 5. _____  | 30. _____ |
| 6. _____  | 31. _____ |
| 7. _____  | 32. _____ |
| 8. _____  | 33. _____ |
| 9. _____  | 34. _____ |
| 10. _____ | 35. _____ |
| 11. _____ | 36. _____ |
| 12. _____ | 37. _____ |
| 13. _____ | 38. _____ |
| 14. _____ | 39. _____ |
| 15. _____ | 40. _____ |
| 16. _____ | 41. _____ |
| 17. _____ | 42. _____ |
| 18. _____ | 43. _____ |
| 19. _____ | 44. _____ |
| 20. _____ | 45. _____ |
| 21. _____ | 46. _____ |
| 22. _____ | 47. _____ |
| 23. _____ | 48. _____ |
| 24. _____ | 49. _____ |
| 25. _____ | 50. _____ |

**Student's Name:** \_\_\_\_\_

**Teacher's Name:** \_\_\_\_\_

1. What is the first level difference for the sequence generated by  $t_n = 6n - 2$ ?

- (A) -4
- (B) -2
- (C) 6
- (D) 12

2. What is  $t_n$  for the sequence  $\{-7, -5, -3, -1, 1, \dots\}$ ?

- (A)  $t_n = -2n - 5$
- (B)  $t_n = -2n - 9$
- (C)  $t_n = 2n - 5$
- (D)  $t_n = 2n - 9$

3. Which table shows a quadratic relationship?

(A)

|     |   |   |   |    |    |
|-----|---|---|---|----|----|
| $x$ | 0 | 1 | 2 | 3  | 4  |
| $y$ | 1 | 3 | 9 | 27 | 81 |

(B)

|     |   |   |    |    |     |
|-----|---|---|----|----|-----|
| $x$ | 0 | 1 | 2  | 3  | 4   |
| $y$ | 1 | 4 | 25 | 82 | 193 |

(C)

|     |   |   |    |    |    |
|-----|---|---|----|----|----|
| $x$ | 0 | 1 | 2  | 3  | 4  |
| $y$ | 1 | 6 | 17 | 34 | 57 |

(D)

|     |   |   |   |    |    |
|-----|---|---|---|----|----|
| $x$ | 0 | 1 | 2 | 3  | 4  |
| $y$ | 1 | 4 | 7 | 10 | 13 |

4. The sequence  $\{2, 9, 28, 65, 126 \dots\}$  is an example of which type of sequence?

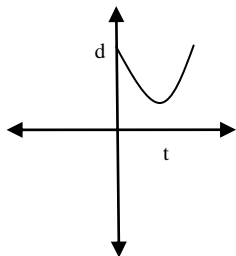
- (A) arithmetic
- (B) cubic
- (C) linear
- (D) quadratic

5. A sequence is defined by the equation  $t_n = 2n + 1$ . For what value of  $n$  is  $t_n = 129$ ?

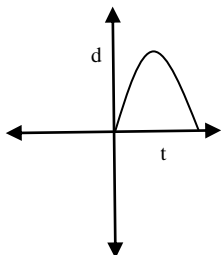
- (A) 64
- (B) 65
- (C) 258
- (D) 259

6. A cart is given a push, rolls up a ramp, stops, and then rolls back down. Which of the following represents the cart's distance from the starting position,  $d$ , over time,  $t$ ?

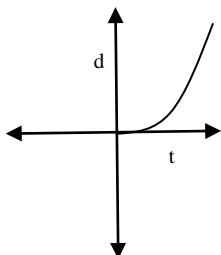
(A)



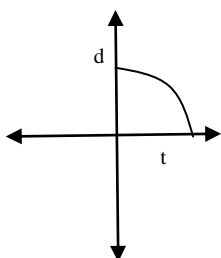
(B)



(C)



(D)



7. What is the vertical stretch of the function  $y = -2x^2 + 3x + 5$ ?

(A)  $-2$

(B)  $-\frac{1}{2}$

(C)  $\frac{1}{2}$

(D)  $2$

8. What value of " $c$ " makes  $x^2 - 18x + c$  a perfect square trinomial?

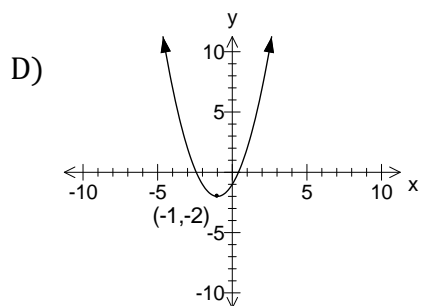
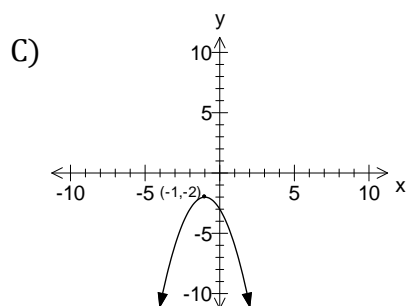
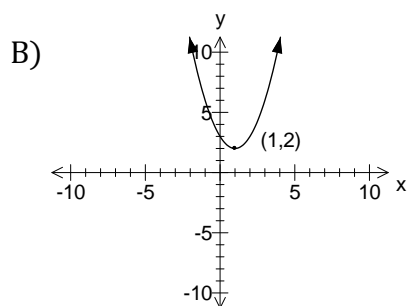
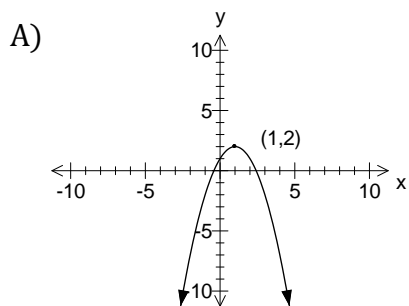
(A)  $-81$

(B)  $18$

(C)  $81$

(D)  $324$

9. Which graph is the transformation of  $y = x^2$  under the mapping rule  $(x, y) \rightarrow (x + 1, -y + 2)$ ?



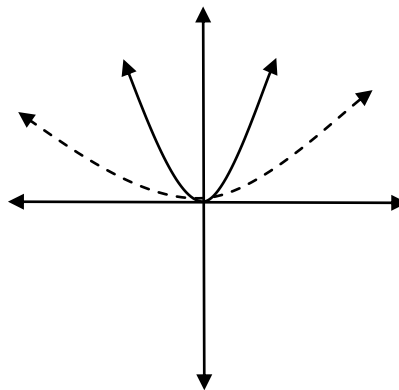
10. What is the general form of  $y = 2(x + 5)^2 - 3$ ?

- (A)  $y = 2x^2 + 47$   
 (B)  $y = 4x^2 + 97$   
 (C)  $y = 2x^2 + 10x + 22$   
 (D)  $y = 2x^2 + 20x + 47$

11. What is the standard form of  $\frac{1}{2}(y + 3) = (x + 1)^2$ ?

- (A)  $-2(y + 3) = (x + 1)^2$   
 (B)  $y = 2x^2 + 4x - 1$   
 (C)  $y = 2(x + 1)^2 - 3$   
 (D)  $y = (x + 1)^2 - 2$

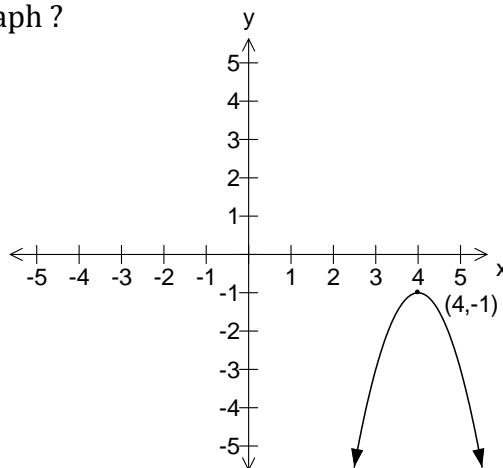
12. The solid graph represents  $y = x^2$ . The dotted graph represents  $y = ax^2$ . What is the **best** possible value of "a"?



- (A)  $-2$   
 (B)  $-\frac{1}{2}$   
 (C)  $\frac{1}{2}$   
 (D)  $2$
13. What is the axis of symmetry for  $y = -3x^2 + 12x - 7$ ?

- (A)  $x = -2$   
 (B)  $x = 2$   
 (C)  $y = -2$   
 (D)  $y = 2$

14. Which equation would produce the following graph ?



- (A)  $-\frac{1}{2}(y - 1) = (x + 4)^2$   
 (B)  $-\frac{1}{2}(y + 1) = (x - 4)^2$   
 (C)  $\frac{1}{2}(y - 1) = (x + 4)^2$   
 (D)  $\frac{1}{2}(y + 1) = (x - 4)^2$
15. Which equation is produced if  $y = x^2$  is translated 3 units to the right, 5 units down, and stretched by a factor of  $\frac{1}{4}$ ?
- (A)  $4(y + 5) = (x - 3)^2$   
 (B)  $4(y - 5) = (x + 3)^2$   
 (C)  $\frac{1}{4}(y + 5) = (x - 3)^2$   
 (D)  $\frac{1}{4}(y - 5) = (x + 3)^2$
16. What are the coordinates of the vertex of the quadratic function having a minimum value of  $-3$  and x-intercepts located at  $(-5, 0)$  and  $(3, 0)$ ?
- (A)  $(-2, -3)$   
 (B)  $(-2, 3)$   
 (C)  $(-1, -3)$   
 (D)  $(-1, 3)$

17. What is the range of  $-\frac{1}{2}(y - 2) = (x + 3)^2$ ?

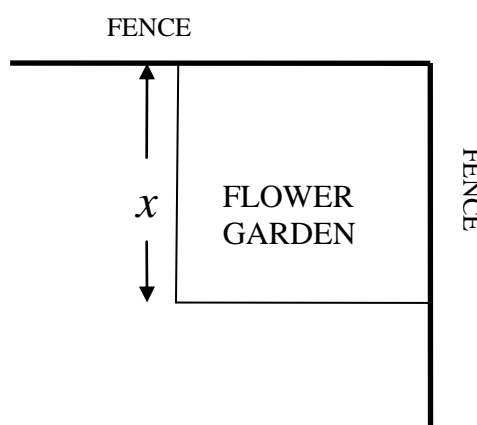
- (A)  $\{y|y \geq -2, y \in R\}$
- (B)  $\{y|y \leq -2, y \in R\}$
- (C)  $\{y|y \geq 2, y \in R\}$
- (D)  $\{y|y \leq 2, y \in R\}$

18. Which function has a maximum value of 3?

- (A)  $y = -2(x + 1)^2 + 3$
- (B)  $y = -2(x - 1)^2 - 3$
- (C)  $y = 2(x + 1)^2 + 3$
- (D)  $y = 2(x - 1)^2 - 3$

19. A rectangular flower garden is to be placed in the corner of a fenced yard. Two of the sides will be made using 4m of rope (the fence will be the other 2 sides). Which equation represents the Area (A) of the garden?

- (A)  $A = x^2$
- (B)  $A = x(x - 4)$
- (C)  $A = x(4 - x)$
- (D)  $A = x(x + 4)$



20. Solve:  $2x^2 + x = 0$

- (A)  $\{0, -2\}$
- (B)  $\{0, -\frac{1}{2}\}$
- (C)  $\{0, \frac{1}{2}\}$
- (D)  $\{0, 2\}$

21. Which equation has x-intercepts of  $-8$  and  $4$ ?

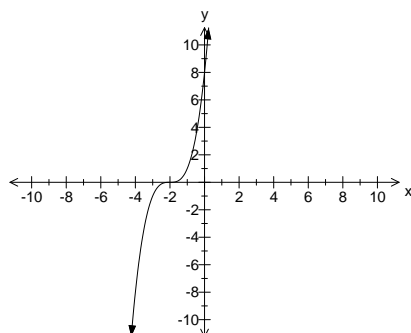
- (A)  $y = x^2 - 12x + 32$
- (B)  $y = x^2 - 4x - 32$
- (C)  $y = x^2 + 4x - 32$
- (D)  $y = x^2 + 12x + 32$

22. Calculate the discriminant of  $2x^2 - 5x - 3 = 0$ .

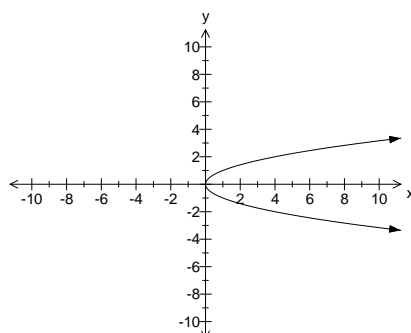
- (A)  $-49$
- (B)  $1$
- (C)  $7$
- (D)  $49$

23. If 3 is one root of the equation  $x^2 + bx + 3 = 0$ , what is the value of  $b$ ?
- (A)  $b = -4$   
 (B)  $b = -3$   
 (C)  $b = 3$   
 (D)  $b = 4$
24. For what value(s) of "k" does the equation  $x^2 + kx + 9 = 0$  have 2 real and equal roots?
- (A)  $k < 6$   
 (B)  $k = 0$   
 (C)  $k = \pm 6$   
 (D)  $k > 6$
25. Which graph represents a quadratic equation with 2 unequal imaginary roots?

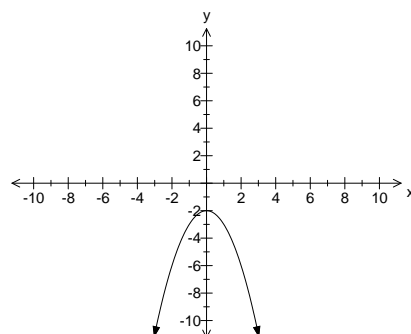
(A)



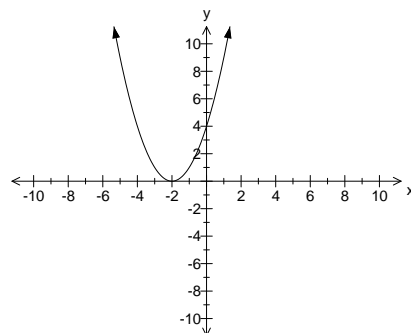
(B)



(C)



(D)





26. What is the **simplest** form of  $\frac{-6 \pm \sqrt{-20}}{2}$ ?

- (A)  $-3 \pm i\sqrt{5}$
- (B)  $-3 \pm i\sqrt{10}$
- (C)  $-3 \pm i\sqrt{20}$
- (D)  $-3 \pm 2i\sqrt{5}$

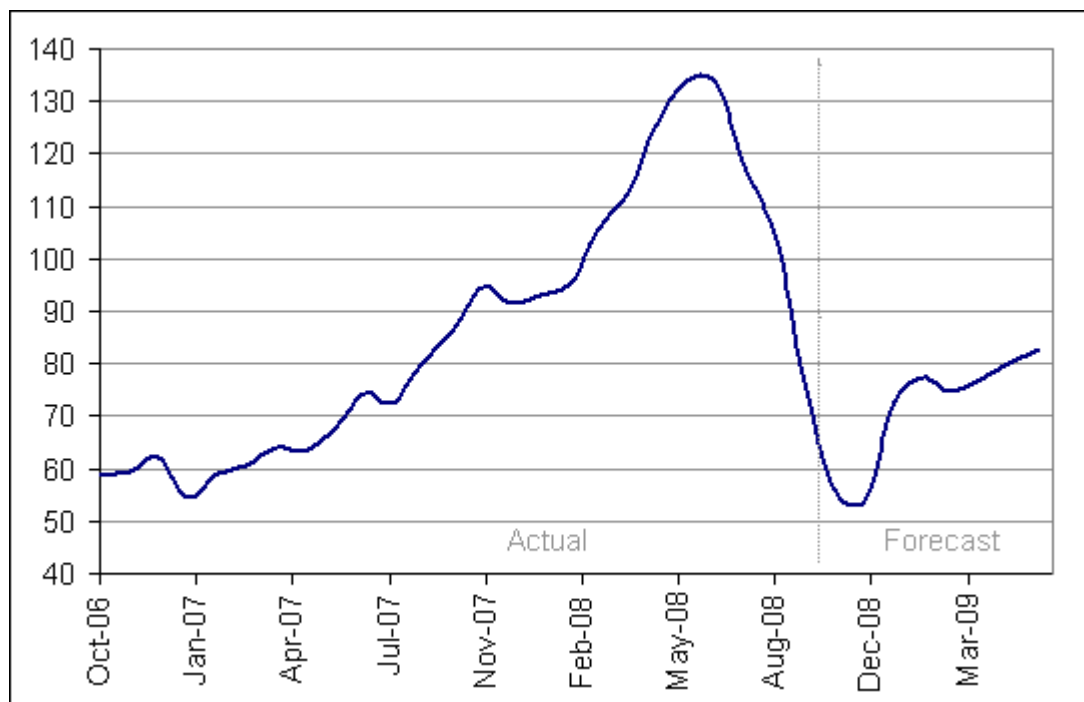
27. What are the roots of  $60 + 5x^2 = 0$ ?

- (A)  $x = \pm 2\sqrt{3}$
- (B)  $x = \pm 3\sqrt{2}$
- (C)  $x = \pm 2i\sqrt{3}$
- (D)  $x = \pm 3i\sqrt{2}$

28. A salmon jumps out of the water and at 1.0 s, its approximate instantaneous rate of change is equal to  $-3.5\text{m/s}$ . Which statement best describes what is happening at this instant?

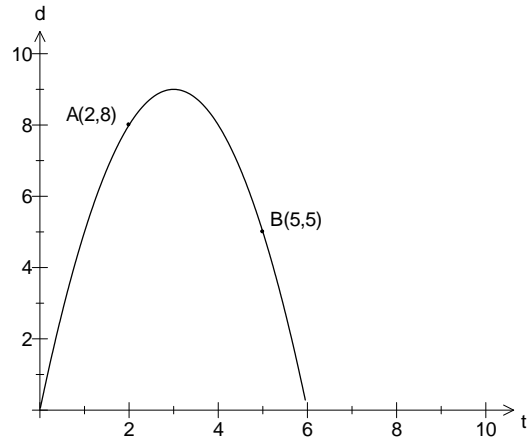
- (A) The salmon is at rest.
- (B) The salmon is gaining height.
- (C) The salmon is at its highest point.
- (D) The salmon is on its way back down.

29. At which point on the graph is there a negative instantaneous rate of change in the price of oil?



- (A) Feb - 08
- (B) May - 08
- (C) Aug - 08
- (D) Dec - 08

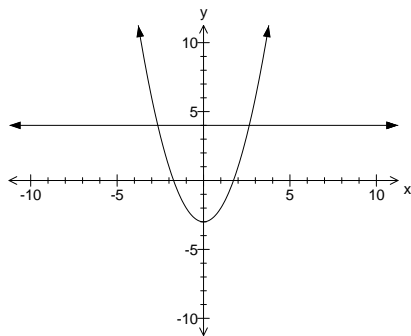
30. What is the average rate of change, in m/s, between 2 and 5 seconds?



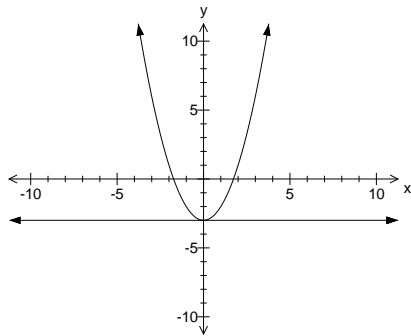
- (A) -3
- (B) -1
- (C) 1
- (D) 3

31. Which graph below illustrates an average rate of change?

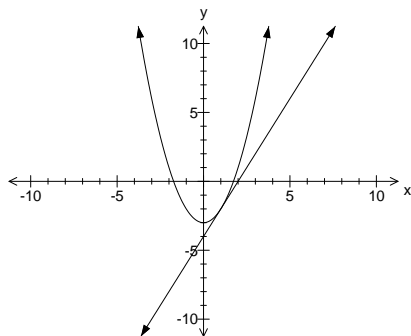
(A)



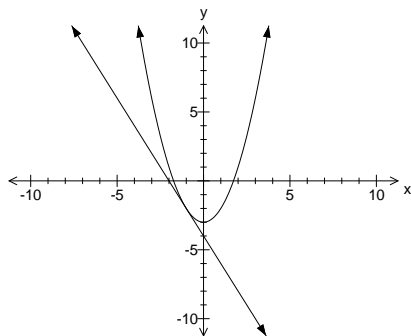
(B)



(C)



(D)



32. What type of sequence is  $\{3, \frac{3}{5}, \frac{3}{25}, \frac{3}{125}, \frac{3}{625}, \dots\}$ ?

- (A) Arithmetic
- (B) Cubic
- (C) Geometric
- (D) Quadratic

33. A house was bought for \$125 000. The house appreciates at a rate of 20% every 8 years. Which function models the value of the house after  $t$  years?

- (A)  $y = 125\,000 (1.20)^{\frac{t}{8}}$
- (B)  $y = 125\,000 (1.20)^{8t}$
- (C)  $y = 125\,000 (0.80)^{\frac{t}{8}}$
- (D)  $y = 125\,000 (0.80)^{8t}$

34. Find  $f(-2)$  if  $f(x) = 5^{-(x+1)}$ ?

- (A)  $-\frac{1}{5}$
- (B)  $\frac{1}{125}$
- (C) 5
- (D) 125

35. Simplify:  $(4^0 + 2^{-3})^{-2}$

- (A)  $\frac{64}{1089}$
- (B)  $\frac{64}{81}$
- (C)  $\frac{81}{64}$
- (D)  $\frac{1089}{64}$

36. Which sequence has a common ratio of 2?

(A)

|       |    |    |    |    |   |
|-------|----|----|----|----|---|
| $n$   | 1  | 2  | 3  | 4  | 5 |
| $t_n$ | -7 | -5 | -3 | -1 | 1 |

(B)

|       |   |    |    |    |    |
|-------|---|----|----|----|----|
| $n$   | 1 | 2  | 3  | 4  | 5  |
| $t_n$ | 9 | 12 | 17 | 24 | 33 |

(C)

|       |   |   |   |               |               |
|-------|---|---|---|---------------|---------------|
| $n$   | 1 | 2 | 3 | 4             | 5             |
| $t_n$ | 4 | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ |

(D)

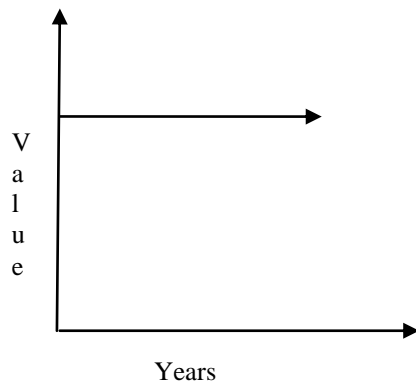
|       |               |               |   |   |   |
|-------|---------------|---------------|---|---|---|
| $n$   | 1             | 2             | 3 | 4 | 5 |
| $t_n$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 |

37. What is the range of  $y - 2 = 2^x$ ?

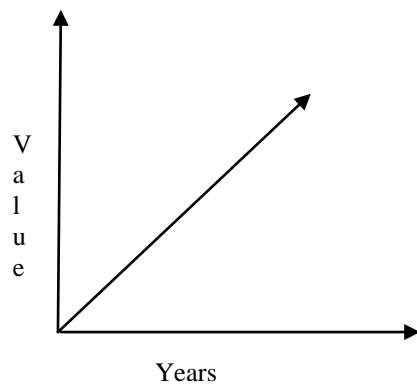
- (A)  $\{x|x \geq 2, x \in R\}$
- (B)  $\{x|x > 2, x \in R\}$
- (C)  $\{y|y \geq 2, x \in R\}$
- (D)  $\{y|y > 2, x \in R\}$

38. Which of the graphs below best models the situation where the value of a car depreciates by 10% annually?

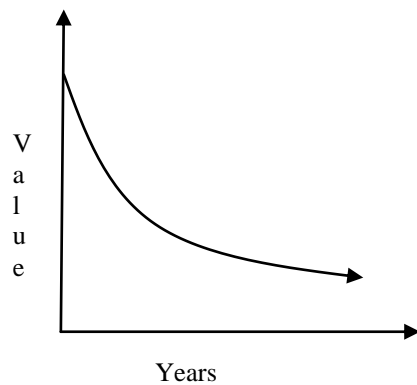
(A)



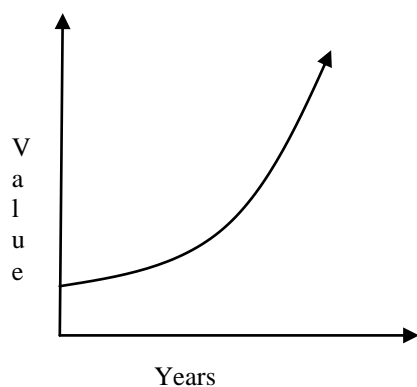
(B)



(C)



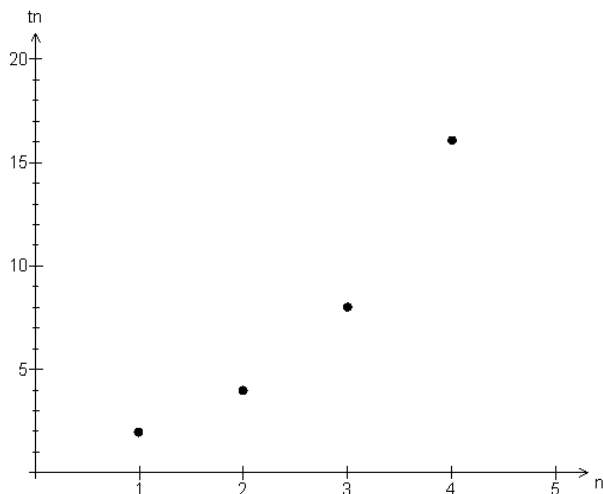
(D)



39. What is the equation of the asymptote of the graph of  $y = 2(3)^x + 5$ ?

- (A)  $y = 2$
- (B)  $y = 3$
- (C)  $y = 5$
- (D)  $y = 7$

40. What is the domain for the graph below?



- (A)  $\{n|n \in \mathbb{N}\}$
- (B)  $\{n|n \geq 0, n \in \mathbb{R}\}$
- (C)  $\{t_n|t_n \in \mathbb{N}\}$
- (D)  $\{t_n|t_n \geq 1, t_n \in \mathbb{R}\}$

41. Which function produces an exponential decay curve?

- (A)  $y = 2(-3)^x$
- (B)  $y = 2\left(-\frac{1}{3}\right)^x$
- (C)  $y = 2\left(\frac{1}{3}\right)^x$
- (D)  $y = 2(3)^x$

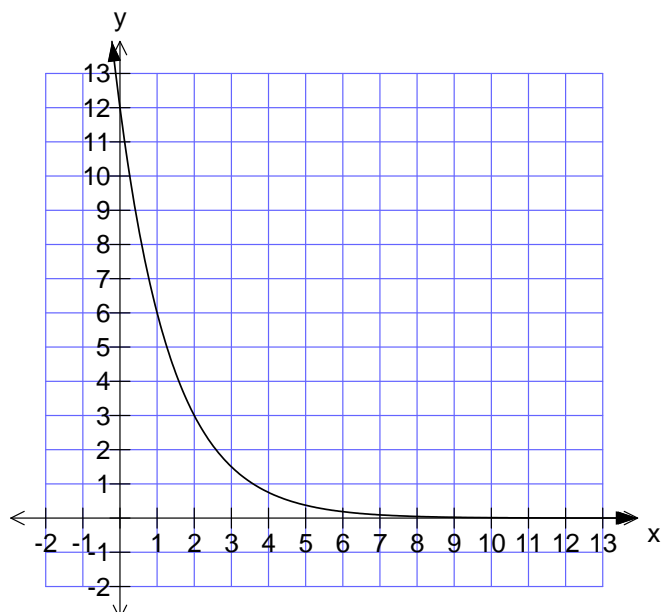
42. Which situation is best modeled by an exponential equation?

- (A) The height of a rock thrown into the air from the time it is thrown to the time it hits the ground
- (B) The height of a seagull over time as it bobs up and down on the ocean.
- (C) The distance a car travels, at a constant speed of  $100 \text{ km/h}$  over time.
- (D) The temperature of a cup of coffee as it cools over time.

43. What is the common ratio between successive terms of the sequence generated by  $t_n = 2\left(\frac{1}{5}\right)^n$ ?

- (A)  $\frac{1}{5}$
- (B)  $\frac{1}{2}$
- (C) 2
- (D) 5

44. Given the graph of  $y = f(x)$ , determine the value of  $f(x)$  when  $x = 2$ .



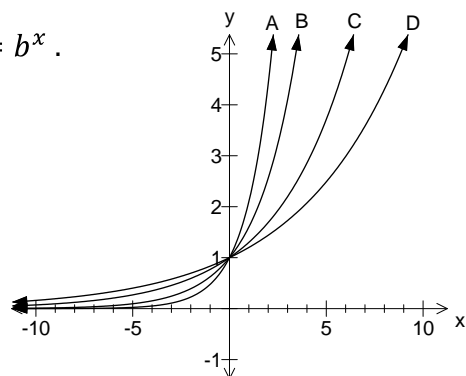
- (A) 1.5  
 (B) 2  
 (C) 3  
 (D) 4
45. The table shows the population,  $N$ , of a culture of bacteria over time,  $t$ . Which function best models the population?

| $t$ | 0    | 6    | 12   | 18    |
|-----|------|------|------|-------|
| $N$ | 1000 | 2500 | 6250 | 15625 |

- (A)  $N = 1000(2.5)^{\frac{t}{6}}$   
 (B)  $N = 1000(2.5)^{6t}$   
 (C)  $N = 1000(2.5)^{\frac{6}{t}}$   
 (D)  $N = 1000(2.5)^{t+6}$
46. Which best describes the function  $y + 4 = 3(5)^x$ ?
- (A) Exponential decay, asymptote  $y = -4$ .  
 (B) Exponential decay, asymptote  $y = 2$ .  
 (C) Exponential growth, asymptote  $y = -4$ .  
 (D) Exponential growth, asymptote  $y = 2$ .
47. What is the  $y$ -intercept of the graph of  $y = 3(4)^{\frac{x}{2}} - 1$ ?

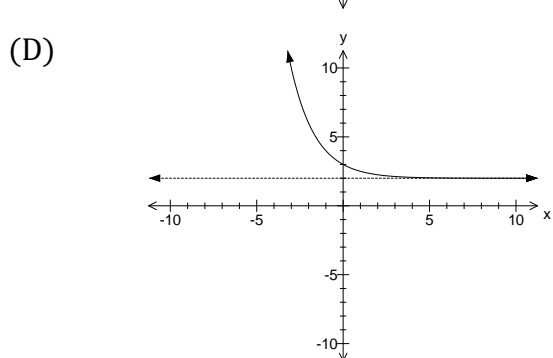
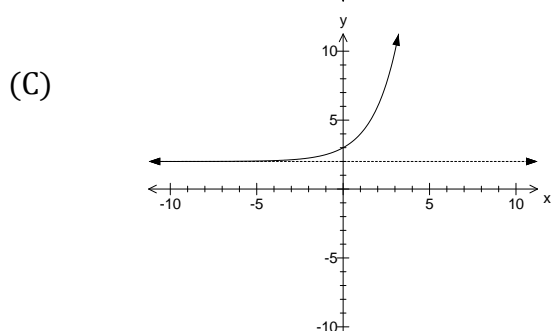
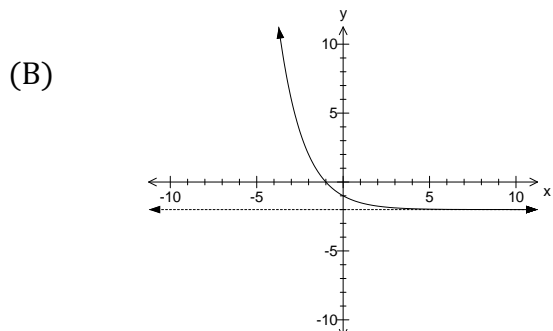
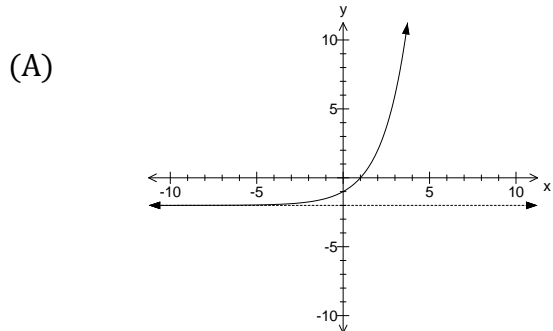
- (A) -1  
 (B) 0  
 (C) 2  
 (D) 11

48. The functions graphed to the right are in the form  $y = b^x$ . Which function has the largest value of  $b$ ?

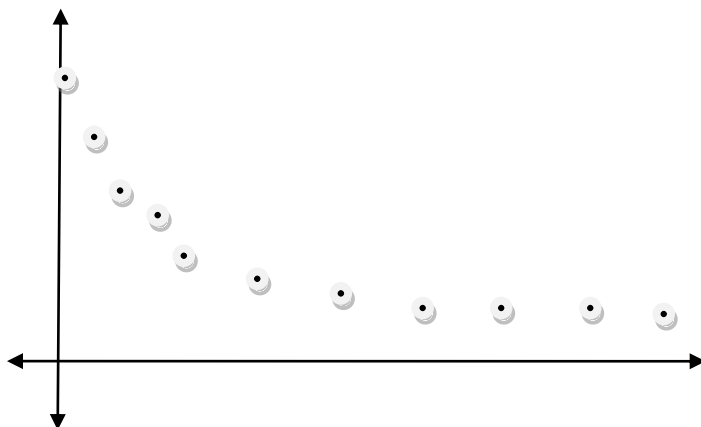


- (A) A  
 (B) B  
 (C) C  
 (D) D

49. Which graph could represent the equation  $y = b^x + 2$ , where  $b > 1$ ?



50. Which equation best models the scatterplot?



- (A)  $y = mx + b$
- (B)  $y = a(x - h)^2 + k$
- (C)  $y = a \cdot b^{\frac{x}{c}} + k$
- (D)  $y = ax^3$

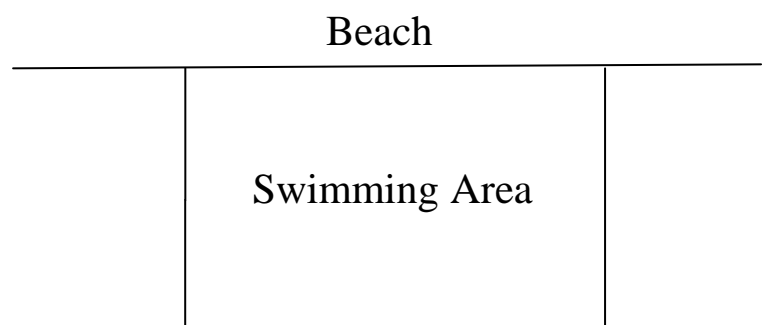
**PART II**  
**Total Value 50%**

Answer **ALL** items in the space provided. Show **ALL** workings.

*value*

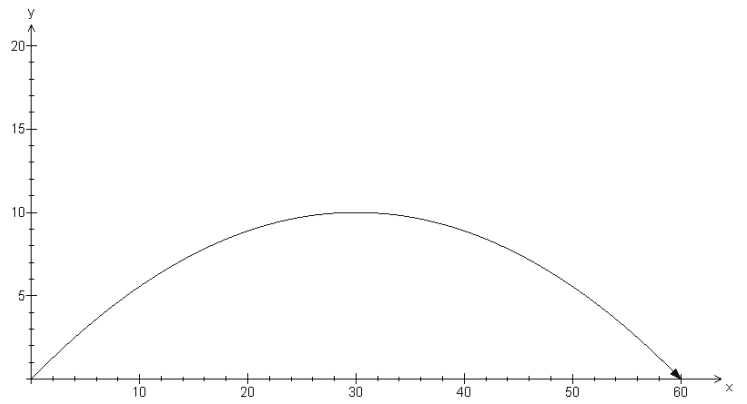
- 4 51. Algebraically determine the **EXACT** roots in simplest form for  $3x(x - 4) = 3$ .

- 4 52. A rectangular area for swimmers is to be made with 120m of rope using the beach as one side, as in the diagram. Algebraically determine the quadratic function which models the swimming area and use it to find the dimensions which produce the maximum area.

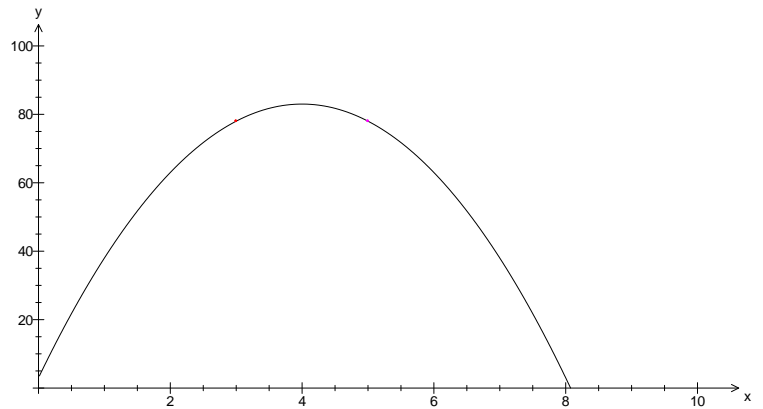




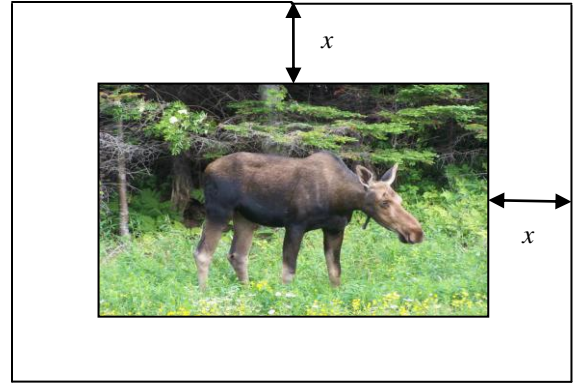
- 4 53. A soccer ball is kicked from the ground and lands on the ground  $60\text{ m}$  away. Algebraically determine the quadratic function representing its path if the maximum height is  $10\text{ m}$ .



- 4 54. A bottle rocket is shot into the air from a  $3\text{ m}$  launch pad, as shown in the diagram. The height of the rocket above the ground, in metres,  $t$  seconds after being shot is approximated by  $h(t) = -5t^2 + 40t + 3$ . Algebraically determine the times when the rocket is at a height of  $78\text{ m}$ .



- 4 55. A photo measures 20cm by 25cm and it has a frame of uniform width, as shown in the diagram. The combined area of the frame and photograph is  $750\text{cm}^2$ . Algebraically determine the width of the frame.



- 4 56. Change the equation  $y = 3x^2 + 30x + 77$  into transformational form.

- 4 57. While on the moon, Neil Armstrong took one giant leap for mankind into a 2m deep crater. His height in metres, is modeled by  $h(t) = -t^2 + 3t + 2$  where  $t$  is the time in seconds. Algebraically determine Mr. Armstrong's maximum height and the time it took to reach this height.

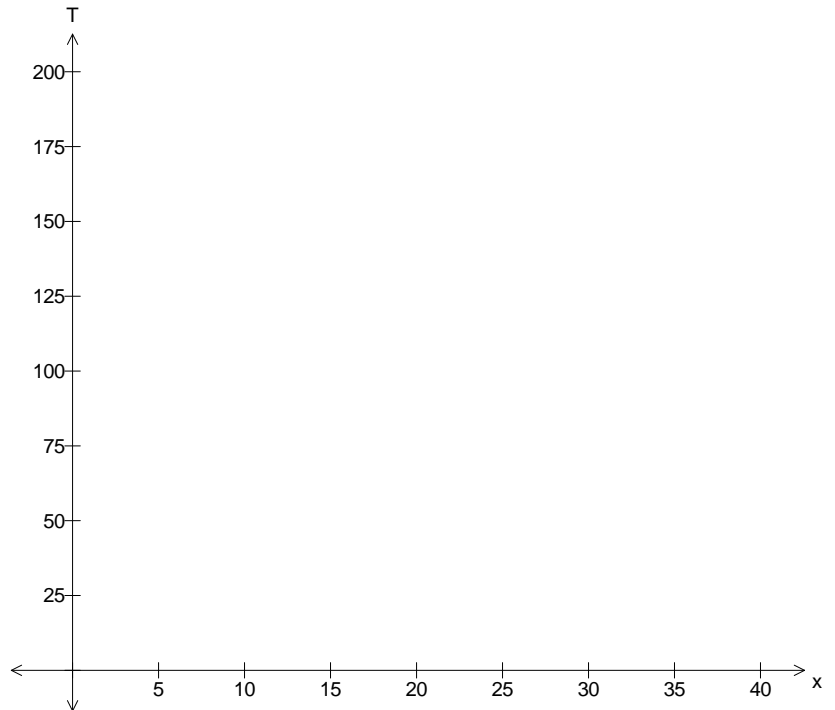
- 4 58. A diver jumps off a 3m springboard. Her height, in metres, above the water,  $t$  seconds after she jumps is modeled by  $h(t) = -5t^2 + 6t + 3$ . Algebraically determine the approximate instantaneous rate of change in her height at 1.0 second and explain what is happening at that instant.

- 4
59. A balloon is being filled with water. Its volume is given by  $V = \frac{4}{3}\pi r^3$ , where  $r$  is the radius of the balloon in centimetres. Determine the average rate of change in the volume of the balloon as its radius changes from 1.0cm to 3.0cm.

- 3
60. Newly discovered radioactive substance Frenchium has a half-life of 34 years. If a sample currently has 550g, how many grams will remain after 119years?

- 3 61. The cooling of a curling iron is modeled by  $T = 150(0.65)^{\frac{x}{2}} + 25$ , where  $T$  is the temperature in degrees Celsius and  $x$  is the time in minutes. Complete the table of values, for the first 30 minutes, then sketch the graph including the asymptote.

| x  | T |
|----|---|
| 0  |   |
| 5  |   |
| 10 |   |
| 15 |   |
| 20 |   |
| 25 |   |
| 30 |   |



- 4 62. Jan invested into a savings account that paid interest at a rate of 4% every 3 years. At the end of 11 years, Jan's account had a balance of \$1558.80. How much money did Jan originally invest?

4

63. The table shows the population of European green crab in Placentia Bay over a period of time. Algebraically determine an equation that models the population of crab,  $P$ , over time,  $t$ . Use the equation to predict the green crab population after 30 years.

|                       |    |     |     |     |     |      |
|-----------------------|----|-----|-----|-----|-----|------|
| Time ( $t$ ) in years | 0  | 3   | 6   | 9   | 12  | 15   |
| Population            | 50 | 100 | 200 | 400 | 800 | 1600 |