

Answer Key Multiple Choice (PART I)

- | | |
|-------|-------|
| 1. C | 26. A |
| 2. D | 27. C |
| 3. C | 28. D |
| 4. B | 29. C |
| 5. A | 30. B |
| 6. B | 31. A |
| 7. D | 32. C |
| 8. C | 33. A |
| 9. A | 34. C |
| 10. D | 35. B |
| 11. C | 36. D |
| 12. C | 37. D |
| 13. B | 38. C |
| 14. B | 39. C |
| 15. A | 40. A |
| 16. C | 41. C |
| 17. D | 42. D |
| 18. A | 43. A |
| 19. C | 44. C |
| 20. B | 45. A |
| 21. C | 46. C |
| 22. D | 47. C |
| 23. A | 48. A |
| 24. C | 49. C |
| 25. C | 50. C |

PART II- Answer Key

51. Algebraically determine the EXACT roots in simplest form for $3x(x - 4) = 3$.

EITHER

$$3x(x - 4) = 3$$

$$3x^2 - 12x = 3 \text{ \{ 0.5 marks \}}$$

$$3x^2 - 12x - 3 = 0 \text{ \{ 0.5 marks \}}$$

$$x^2 - 4x - 1 = 0 \text{ (divide through by 3)}$$

$$a = 1, \quad b = -4, \quad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} \text{ \{ 1 mark \}}$$

$$x = \frac{4 \pm \sqrt{16 - (-4)}}{2} \text{ \{ 0.5 marks \}}$$

$$x = \frac{4 \pm \sqrt{20}}{2} \text{ \{ 0.5 marks \}}$$

$$x = \frac{4 \pm 2\sqrt{5}}{2} \text{ \{ 0.5 marks \}}$$

$$x = 2 \pm \sqrt{5} \text{ \{ 0.5 marks \}}$$

OR

$$3x(x - 4) = 3$$

$$3x^2 - 12x = 3$$

$$3x^2 - 12x - 3 = 0$$

$$a = 3, \quad b = -12, \quad c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-3)}}{2(3)}$$

$$x = \frac{12 \pm \sqrt{144 - (-36)}}{6}$$

$$x = \frac{12 \pm \sqrt{180}}{6}$$

$$x = \frac{12 \pm 6\sqrt{5}}{6}$$

$$x = 2 \pm \sqrt{5}$$

52. A rectangular area for swimmers is to be made with 120m of rope using the beach as one side, as in the diagram. Algebraically determine the quadratic function which models the swimming area and use it to find the dimensions which produce the maximum area.

Either

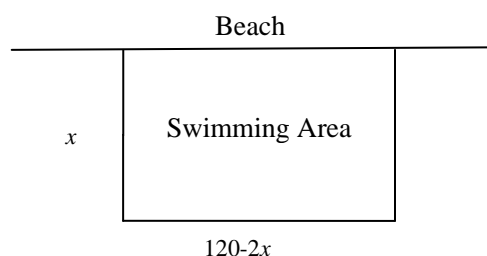
$$A = x(120 - 2x) \text{ \{ 1.5 marks \}}$$

$$A = 120x - 2x^2 \text{ \{ 0.5 marks \}}$$

$$x = \frac{-b}{2a} = \frac{-120}{2(-2)} = \frac{-120}{-4} = 30 \text{ \{ 1 mark \}}$$

The sides are 30 and $120 - 2(30) = 60$ { 1 mark }

The dimensions are 30m by 60m



Or

$$A = x(120 - 2x) \text{ \{ 1.5 marks \}}$$

$$A = 120x - 2x^2 \text{ \{ 0.5 marks \}}$$

$$A = -2(x^2 - 60x)$$

$$A = -2(x^2 - 60x + 900) + 1800$$

$$A = -2(x - 30)(x - 30) + 1800$$

$$A = -2(x - 30)^2 + 1800 \text{ \{ 1 mark \}}$$

Vertex (30, 1800)

The sides are 30 and $120 - 2(30) = 60$ { 1 mark }

The dimensions are 30m by 60m

53. A soccer ball is kicked from the ground and lands on the ground 60m away. Algebraically determine the quadratic function representing its path if the maximum height is 10m.

Vertex is (30,10) { 1 mark }

$$y = a(x - h)^2 + k$$

$$y = a(x - 30)^2 + 10 \text{ { 0.5 marks } }$$

Substitute in (0,0)

$$0 = a(0 - 30)^2 + 10 \text{ { 1 mark } }$$

$$0 = a(-30)^2 + 10$$

$$0 = 900a + 10$$

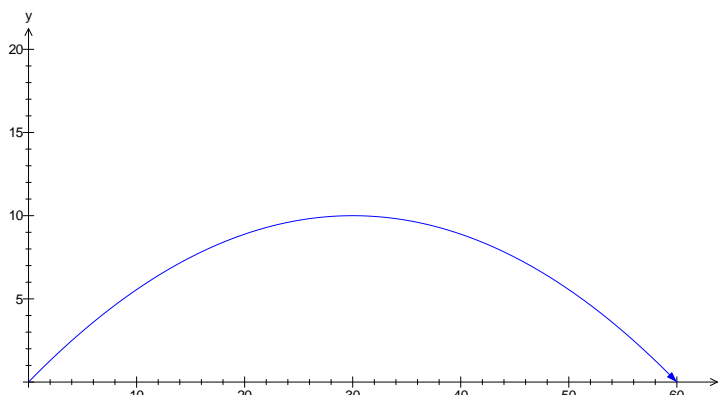
$$-10 = 900a$$

$$\frac{-10}{900} = a$$

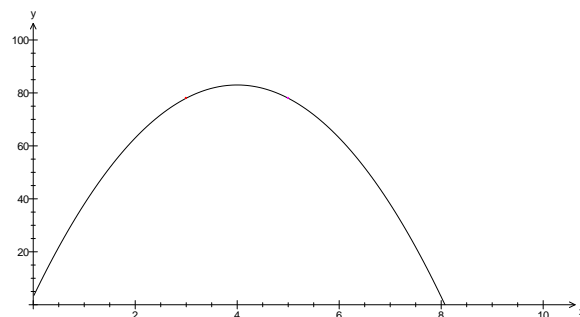
$$-\frac{1}{90} = a \text{ { 1 mark } }$$

The equation is :

$$y = -\frac{1}{90}(x - 30)^2 + 10 \text{ { 0.5 marks } }$$



54. A bottle rocket is shot into the air from a 3m launch pad, as shown in the diagram. The height of the rocket above the ground, in metres, t seconds after being shot is approximated by $h(t) = -5t^2 + 40t + 3$. Algebraically determine the times when the rocket is at a height of 78m.



Either

$$-5t^2 + 40t + 3 = 78 \text{ { 0.5 marks } }$$

$$-5t^2 + 40t - 75 = 0 \text{ { 0.5 marks } }$$

$$t^2 - 8t + 15 = 0 \text{ (divide through by -5)}$$

$$(t - 3)(t - 5) = 0 \text{ { 2 marks } }$$

$$t = 3 \text{ or } t = 5 \text{ { 1 mark } }$$

The rocket is at a height of 78m after 3 seconds and again after 5 seconds.

OR

$$-5t^2 + 40t + 3 = 78 \text{ { 0.5 marks } }$$

$$-5t^2 + 40t - 75 = 0 \text{ { 0.5 marks } }$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(-5)(-75)}}{2(-5)} \text{ { 1 mark } }$$

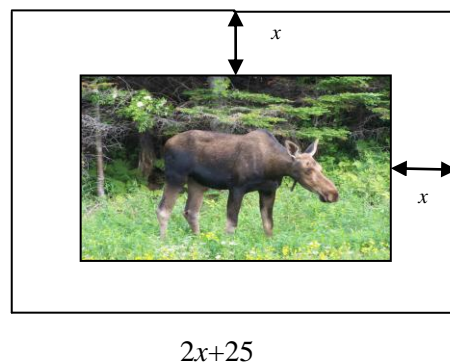
$$t = \frac{-40 \pm \sqrt{100}}{-10} \text{ { 0.5 marks } }$$

$$t = \frac{-40 \pm 10}{-10} \text{ { 0.5 marks } }$$

$$t = 4 \pm 1 \text{ { 0.5 marks } }$$

$$t = 3 \text{ or } t = 5 \text{ { 0.5 marks } }$$

55. A photo measures 20cm by 25cm and it has a frame of uniform width, as shown in the diagram. The combined area of the frame and photograph is 750cm^2 . Algebraically determine the width of the frame.



$$\text{Area} = (2x + 20)(2x + 25) \{ 1 \text{ mark} \}$$

$$\text{Area} = 4x^2 + 50x + 40x + 500$$

$$750 = 4x^2 + 90x + 500$$

$$0 = 4x^2 + 90x - 250 \{ 1 \text{ mark} \}$$

$$a = 4, \quad b = 90, \quad c = -250$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-90 \pm \sqrt{90^2 - 4(4)(-250)}}{2(4)} \{ 1 \text{ mark} \}$$

$$x = \frac{-90 \pm \sqrt{12100}}{8}$$

$$x = \frac{-90 \pm 110}{8}$$

$$x = \frac{-90 - 110}{8} = \frac{-200}{8} = -25 \text{ Which is an inadmissible value length. } \{ 0.5 \text{ marks} \}$$

$$x = \frac{-90 + 110}{8} = \frac{20}{8} = 2.5 \{ 0.5 \text{ marks} \}$$

56. Change the equation $y = 3x^2 + 30x + 77$ into transformational form.

Either

$$y = 3x^2 + 30x + 77$$

$$y = 3(x^2 + 10x) + 77 \{ 1 \text{ mark} \}$$

$$y = 3(x^2 + 10x + 25) + 77 - 75 \{ 1 \text{ mark} \}$$

$$y = 3(x + 5)^2 + 2 \{ 1 \text{ mark} \}$$

$$\frac{1}{3}(y - 2) = (x + 5)^2 \text{ is the equation in transformational form } \{ 1 \text{ mark} \}$$

Or

$$y = 3x^2 + 30x + 77$$

$$a = 3 \{ 1 \text{ mark} \}$$

$$h = \frac{-b}{2a} = \frac{-30}{2(3)} = \frac{-30}{6} = -5 \{ 1 \text{ mark} \}$$

$$k = 3(-5)^2 + 30(-5) + 77$$

$$k = 2 \{ 1 \text{ mark} \}$$

$$\text{Vertex } (-5, 2)$$

$$\frac{1}{3}(y - 2) = (x + 5)^2 \text{ is the equation in transformational form } \{ 1 \text{ mark} \}$$

57. While on the moon, Neil Armstrong took one giant leap for mankind into a 2m deep crater. His height in metres, is modeled by $h(t) = -t^2 + 3t + 2$ where t is the time in seconds. Algebraically determine Mr. Armstrong's maximum height and the time it took to reach this height.

$$h(t) = -t^2 + 3t + 2$$

$$t_v = \frac{-b}{2a} = \frac{-3}{2(-1)} = 1.5 \{ 1.5 \text{ mark} \}$$

$$h(1.5) = -(1.5)^2 + 3(1.5) + 2 \{ 1 \text{ mark} \}$$

$$h(1.5) = 4.25 \{ 0.5 \text{ mark} \}$$

The maximum height of 4.25m { 0.5 marks }
was reached after 1.5 seconds { 0.5 marks }

58. A diver jumps off a 3m springboard. Her height, in metres, above the water, t seconds after she jumps is modeled by $h(t) = -5t^2 + 6t + 3$. Algebraically determine the approximate instantaneous rate of change in her height at 1.0 second and explain what is happening at that instant.

$$IROC = \frac{h(t_2) - h(t_1)}{t_2 - t_1}$$

For $t_1 = 0.9$, $h(0.9) = -5(0.9)^2 + 6(0.9) + 3 = 4.35 \{ 1 \text{ mark} \}$
For $t_2 = 1.1$, $h(1.1) = -5(1.1)^2 + 6(1.1) + 3 = 3.55 \{ 1 \text{ mark} \}$

$$IROC = \frac{3.55 - 4.35}{0.2} = -4 \{ 1 \text{ mark} \}$$

The approximate instantaneous rate of change is -4 m/s
The diver is (falling, on the way down, past the highest point etc.) { 1 mark }

59. A balloon is being filled with water. It's volume is given by the equation $V = \frac{4}{3}\pi r^3$, where r is the radius of the balloon in centimetres. Determine the average rate of change in the volume of the balloon as its radius changes from 1.0cm to 3.0cm.

$$AROC = \frac{V(r_2) - V(r_1)}{r_2 - r_1}$$

For $r_1 = 1.0$, $V(1.0) = \frac{4}{3}\pi(3.0)^3 = 113.10 \{ 1 \text{ mark} \}$
For $r_2 = 3.0$, $V(3.0) = \frac{4}{3}\pi(1.0)^3 = 4.19 \{ 1 \text{ mark} \}$

$$AROC = \frac{113.10 - 4.19}{2} = 54.5 \{ 2 \text{ mark} \}$$

The average rate of change is $54.5 \text{ cm}^3/\text{cm}$

60. Newly discovered radioactive substance Frenchium has a half-life of 34 years. If a sample currently has 550g, how many grams will remain after 119 years?

$$y = a \cdot b^{x/c}$$

$$y = 550 \cdot 0.5^{119/34} \{ 2 \text{ mark} \}$$

$$y = 550 \cdot 0.5^{3.5}$$

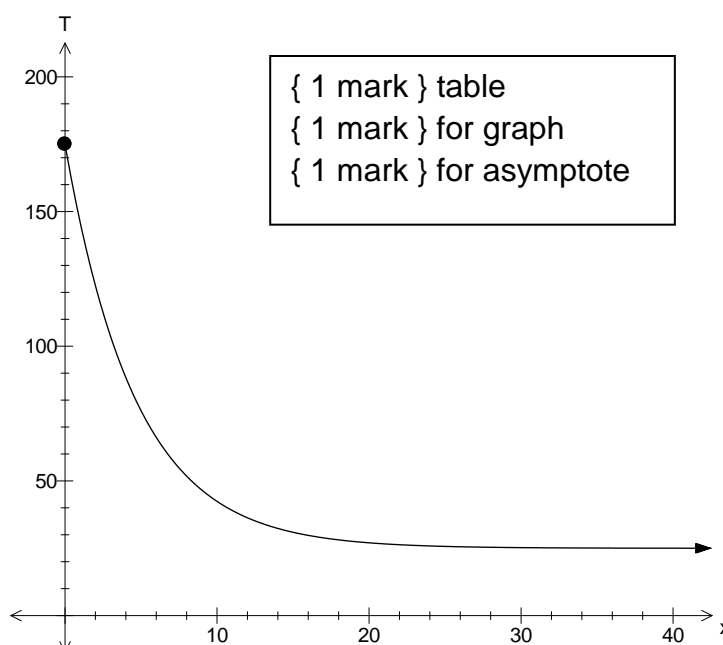
$$y = 550 \cdot (.08838)$$

$$y = 48.6g \{ 1 \text{ mark} \}$$

There will be 48.6g remaining

61. The cooling of a curling iron is modeled by $T = 150(0.65)^{\frac{x}{2}} + 25$, where T is the temperature in degrees Celsius and x is the time in minutes. Complete the table of values, for the first 30 minutes, then sketch the graph including the asymptote.

x	T
0	175
5	76.0
10	42.4
15	30.9
20	27.0
25	25.7
30	25.2



62. Jan invested into a savings account that paid interest at a rate of 4% every 3 years. At the end of 11 years, Jan's account had a balance of \$1558.80. How much money did Jan originally invest?

$$y = a \cdot b^{x/c}$$

$$1558.80 = a \cdot (1.04)^{11/3} \{ 2 \text{ mark} \}$$

$$1558.80 = a \cdot (1.1546638) \{ 1 \text{ mark} \}$$

$$a = \frac{1558.80}{1.1546638} \{ 0.5 \text{ mark} \}$$

$$a = 1350.00 \{ 0.5 \text{ mark} \}$$

The initial amount invested was \$1350.

63. The table shows the population of European green crab in Placentia Bay over a period of time. Algebraically determine an equation that models the population of crab, P , over time, t . Use the equation to predict the green crab population after 30 years.

Time (t) in years	0	3	6	9	12	15
Population	50	100	200	400	800	1600

$$y = a \cdot b^{x/c}$$

$$b = 2 \{ 1 \text{ mark} \}$$

$$c = 3 \{ 0.5 \text{ mark} \}$$

$$a = 50 \{ 0.5 \text{ mark} \}$$

The equation is $y = 50 \cdot 2^{x/3}$ { 1 mark }

After 30 years $y = 50 \cdot 2^{30/3}$ or $y = 50 \cdot 2^{10} = 51200$

After 30 years there will be about 51200 crab. { 1 mark }