

**STUDENT'S NAME**

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**TEACHER'S NAME**

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***DO NOT OPEN THIS EXAMINATION PAPER UNTIL  
YOU ARE TOLD BY THE SUPERVISOR TO BEGIN***



**MIDYEAR EXAMINATION  
MATHEMATICS 3204**

**January 23, 2007**

Value 100 Marks

Time: 3 hours

***General Instructions***

1. Students are required to do **ALL** items.
2. The examination consists of the following parts:  

|                 |                      |            |
|-----------------|----------------------|------------|
| <b>PART I:</b>  | Selected Response    | Value: 50% |
| <b>PART II:</b> | Constructed Response | Value: 50% |
3. A self-powered calculator may be used for calculations and to obtain special values.
4. Answers to **PART I** items are to be shaded on the computer scorable answer sheet. If a second sheet is provided for **PART I** items, letters should be clearly written and this sheet ***stapled to the front*** of the examination paper.
5. For **PART II** items, students are reminded to show all necessary steps and calculations as credit may be given for incomplete or for partially correct solutions. Correct answers without calculations will not merit full marks.

***Student Checklist***

***The following items are your responsibility. Please ensure that they are completed.***

- Check that you are doing the correct exam.
- Write your name on the top of this page and on any answer sheet.
- Check that the bubble sheet is adequately shaded.
- Check this exam to ensure that there are no missing pages.
- At the end of the examination period check that you have completed or at least attempted **ALL** items.

## Teacher Grading Sheet ( PART I )

- |           |           |
|-----------|-----------|
| 1. _____  | 26. _____ |
| 2. _____  | 27. _____ |
| 3. _____  | 28. _____ |
| 4. _____  | 29. _____ |
| 5. _____  | 30. _____ |
| 6. _____  | 31. _____ |
| 7. _____  | 32. _____ |
| 8. _____  | 33. _____ |
| 9. _____  | 34. _____ |
| 10. _____ | 35. _____ |
| 11. _____ | 36. _____ |
| 12. _____ | 37. _____ |
| 13. _____ | 38. _____ |
| 14. _____ | 39. _____ |
| 15. _____ | 40. _____ |
| 16. _____ | 41. _____ |
| 17. _____ | 42. _____ |
| 18. _____ | 43. _____ |
| 19. _____ | 44. _____ |
| 20. _____ | 45. _____ |
| 21. _____ | 46. _____ |
| 22. _____ | 47. _____ |
| 23. _____ | 48. _____ |
| 24. _____ | 49. _____ |
| 25. _____ | 50. _____ |

**Student's Name:**

\_\_\_\_\_

**Teacher's Name:**

\_\_\_\_\_

**PART I**  
**Total Value: 50%**

Answer all items. Shade the letter of the correct answer on the computer scorable answer sheet and write the correct letter in the answer sheet as directed by your teacher. All items on Part I have a value of one mark.

1. Which of the following represents a cubic sequence?

- (A)  $t_n = 3n + 1$
- (B)  $t_n = 2n^2 + n^3 + 1$
- (C)  $t_n = 3n^3 - 2n^4 + n + 4n^2$
- (D)  $t_n = 4\left(\frac{1}{2}\right)^n$

2. Which table below describes a quadratic relationship?

(A)

|     |   |   |   |   |   |
|-----|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 |
| $y$ | 0 | 2 | 4 | 6 | 8 |

(B)

|     |    |    |    |    |    |
|-----|----|----|----|----|----|
| $x$ | 0  | 2  | 1  | 3  | 4  |
| $y$ | 20 | 23 | 28 | 35 | 44 |

(C)

|     |   |   |   |    |    |
|-----|---|---|---|----|----|
| $x$ | 0 | 1 | 2 | 3  | 4  |
| $y$ | 0 | 3 | 8 | 15 | 24 |

(D)

|     |   |   |   |   |    |
|-----|---|---|---|---|----|
| $x$ | 0 | 1 | 2 | 3 | 4  |
| $y$ | 0 | 2 | 4 | 8 | 16 |

3. Which rule describes the sequence  $\{1, -2, -5, -8, -11, \dots\}$ ?

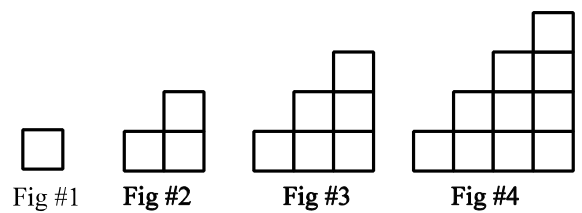
- (A)  $t_n = -3n - 2$
- (B)  $t_n = -3n + 1$
- (C)  $t_n = -3n + 4$
- (D)  $t_n = 3n + 1$

4. Given  $t_n = -2n^2 + 5$ , what is the value of  $t_6$ ?

- (A) -139
- (B) -67
- (C) -19
- (D) 149

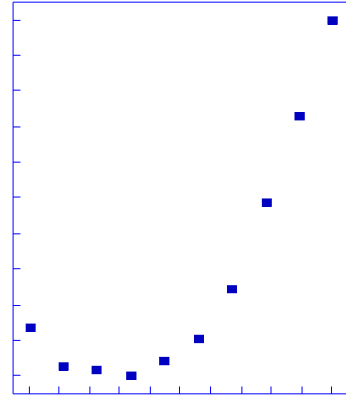
5. The figures in the diagram below were created using toothpicks. Fig# 1 has four toothpicks. This pattern is quadratic. What is the second level difference ( $D_2$ ) ?

- (A) 2  
 (B) 4  
 (C) 6  
 (D) 8



6. Which equation best models the scatter plot?

- (A)  $y = mx + b$   
 (B)  $x = a$   
 (C)  $y = ax^2 + bx + c$   
 (D)  $y = a \cdot b^x$



7. Which type of function models this data?

|     |   |   |   |    |    |
|-----|---|---|---|----|----|
| $x$ | 1 | 2 | 3 | 4  | 5  |
| $y$ | 9 | 5 | 9 | 21 | 41 |

- (A) cubic  
 (B) exponential  
 (C) linear  
 (D) quadratic

8. Which equation has roots  $x = -\frac{1}{2}$  and  $x = 3$  ?

- (A)  $(x + 2)(x + 3) = 0$   
 (B)  $(x - 2)(x - 3) = 0$   
 (C)  $(2x + 1)(x + 3) = 0$   
 (D)  $(2x + 1)(x - 3) = 0$

9. Which equation is the transformational form of  $y = 3(x + 5)^2 - 7$  ?

- (A)  $-3(y - 7) = (x + 5)^2$   
 (B)  $-3(y + 7) = (x + 5)^2$   
 (C)  $\frac{1}{3}(y - 7) = (x + 5)^2$   
 (D)  $\frac{1}{3}(y + 7) = (x + 5)^2$

10. What is the  $y$  - intercept of the quadratic function  $4(y - 2) = (x + 4)^2$  ?

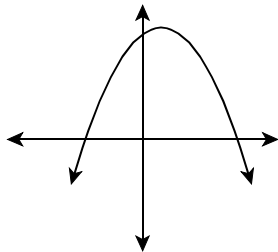
- (A)  $(0, -4)$
- (B)  $(0, 2)$
- (C)  $(0, 6)$
- (D)  $(0, 66)$

11. Which value of “ $c$ ” makes  $x^2 - 7x + c$  a perfect square?

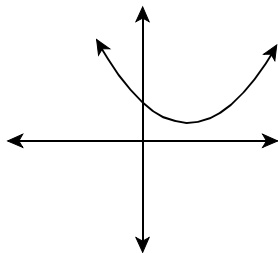
- (A)  $-49$
- (B)  $-\frac{49}{4}$
- (C)  $\frac{49}{4}$
- (D)  $49$

12. Which of the following could have a discriminant value of  $-3$ ?

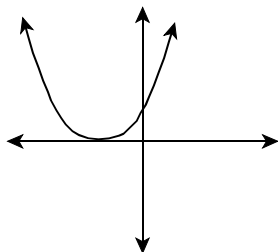
A)



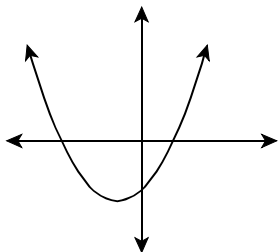
B)



C)

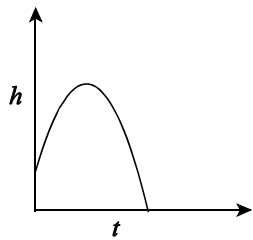


D)

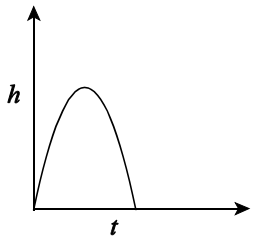


13. What is the equation of the axis of symmetry of  $f(x) = (x + 5)^2 - 7$ ?
- (A)  $f(x) = -7$   
 (B)  $x = -5$   
 (C)  $f(x) = 7$   
 (D)  $x = 5$
14. What is the general form of  $y = -2(x - 5)^2 + 39$ ?
- (A)  $y = -2x^2 - 11$   
 (B)  $y = -2x^2 - 64$   
 (C)  $y = -2x^2 + 20x - 11$   
 (D)  $y = -2x^2 + 20x - 89$
15. What is the value of the discriminant of  $x^2 - 4x - 3 = 0$ ?
- (A) -28  
 (B) -4  
 (C) 4  
 (D) 28
16. Which graph could model the following situation: “The height of a snowball thrown into the air from a height of 1.5 m, over a period of time”?

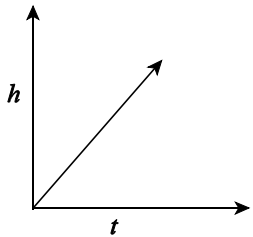
(A)



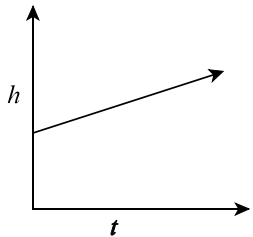
(B)



(C)

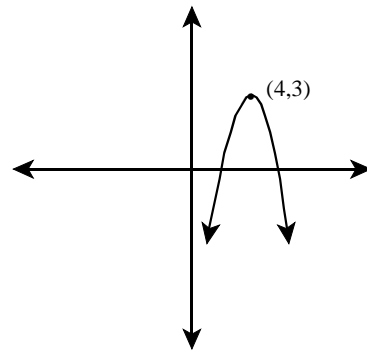


(D)



17. Which maps the function  $y = x^2$  onto the given parabola?

- (A)  $(x, y) \rightarrow (x + 4, -y + 3)$
- (B)  $(x, y) \rightarrow (-x - 4, y - 3)$
- (C)  $(x, y) \rightarrow (x - 4, -y - 3)$
- (D)  $(x, y) \rightarrow (-x + 4, y + 3)$



18. Simplify:  $3 \pm \sqrt{-81}$

- (A)  $-6$  and  $12$
- (B)  $-6i$  and  $12$
- (C)  $-3 + 9i$  and  $-3 + 9i$
- (D)  $3 + 9i$  and  $3 - 9i$

19. What is the nature of the roots of a quadratic function with  $b^2 - 4ac > 0$ ?

- (A) Two unequal real roots
- (B) Two equal real roots
- (C) Two unequal imaginary roots
- (D) Two equal imaginary roots

20. Solve:  $x^2 - 3x + 1 = 0$

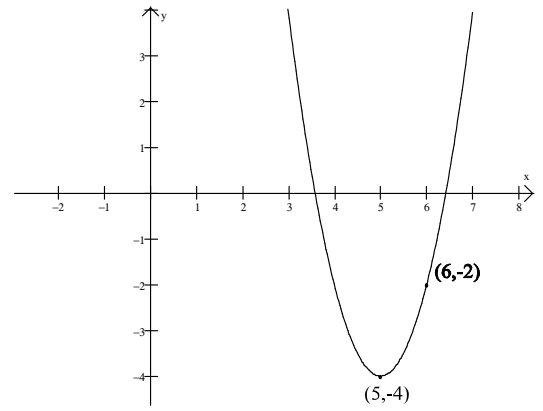
- (A)  $\frac{-3 \pm \sqrt{5}}{2}$
- (B)  $\frac{-3 \pm i\sqrt{13}}{2}$
- (C)  $\frac{3 \pm \sqrt{5}}{2}$
- (D)  $\frac{3 \pm \sqrt{13}}{2}$

21. The function  $y = x^2$  is transformed using the mapping rule  $(x, y) \rightarrow (x - 5, 2y + 3)$ . What is the corresponding equation?

- (A)  $\frac{1}{2}(y - 3) = (x + 5)^2$
- (B)  $\frac{1}{2}(y + 3) = (x - 5)^2$
- (C)  $2(y - 3) = (x + 5)^2$
- (D)  $2(y + 3) = (x - 5)^2$

22. What transformation of  $y = x^2$  results in the graph?

- (A) A vertical translation of 4 up, a horizontal translation of 5 right and a vertical stretch of 2.
- (B) A vertical translation of 4 down, a horizontal translation of 5 right and a vertical stretch of 2.
- (C) A vertical translation of 4 up, a horizontal translation of 5 right and a vertical stretch of  $\frac{1}{2}$ .
- (D) A vertical translation of 4 down, a horizontal translation of 5 right and a vertical stretch of  $\frac{1}{2}$ .



23. Which has the greatest vertical stretch factor when compared to  $y = x^2$ ?

- (A)  $-\frac{1}{2}(y + 2) = x^2$
- (B)  $-\frac{1}{3}(y + 2) = x^2$
- (C)  $(y + 2) = x^2$
- (D)  $4(y + 3) = x^2$

24. What are the  $x$ -intercepts of  $y = 4x^2 - 80$ ?

- (A)  $\pm 2\sqrt{5}$
- (B)  $\pm 4\sqrt{5}$
- (C)  $\pm 5\sqrt{2}$
- (D)  $\pm 2i\sqrt{5}$

25. What are the roots of  $x^2 + 8 = -1$ ?

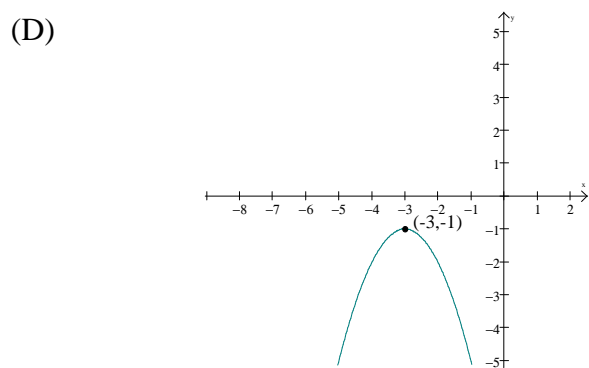
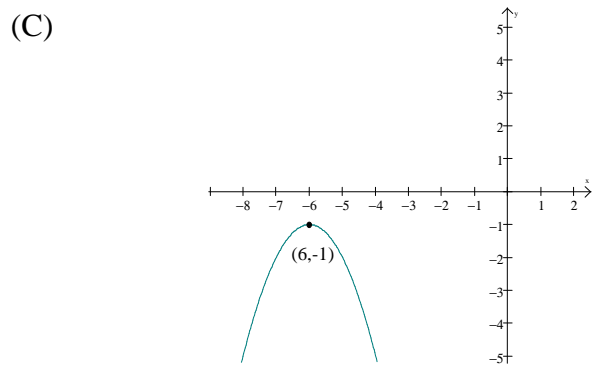
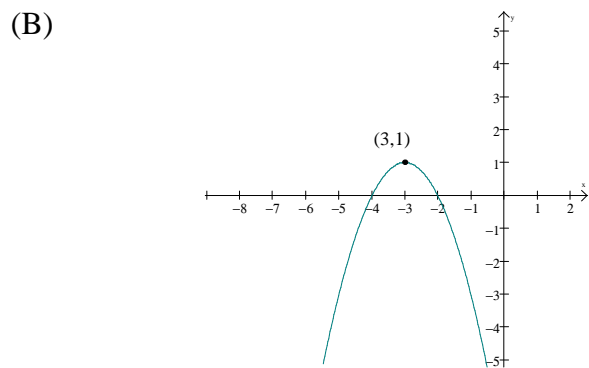
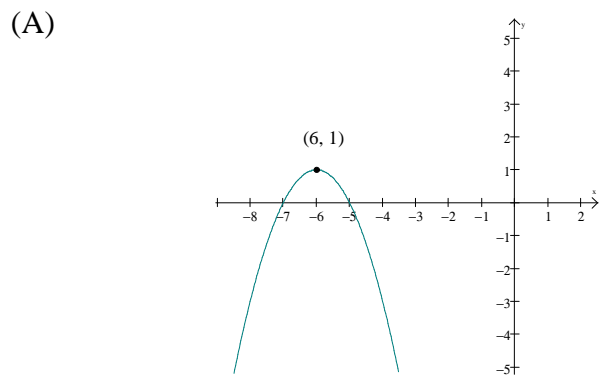
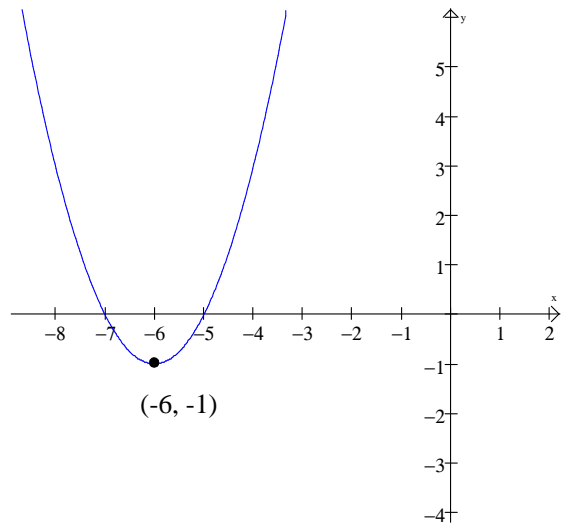
- (A)  $\pm 8$
- (B)  $\pm 3$
- (C)  $\pm i\sqrt{7}$
- (D)  $\pm 3i$

26. What is the range of  $-\frac{1}{2}(y - 1) = (x - 3)^2$ ?

- (A)  $\{y \mid y \leq -1; y \in R\}$
- (B)  $\{y \mid y \geq -1; y \in R\}$
- (C)  $\{y \mid y \leq 1; y \in R\}$
- (D)  $\{y \mid y \geq 1; y \in R\}$

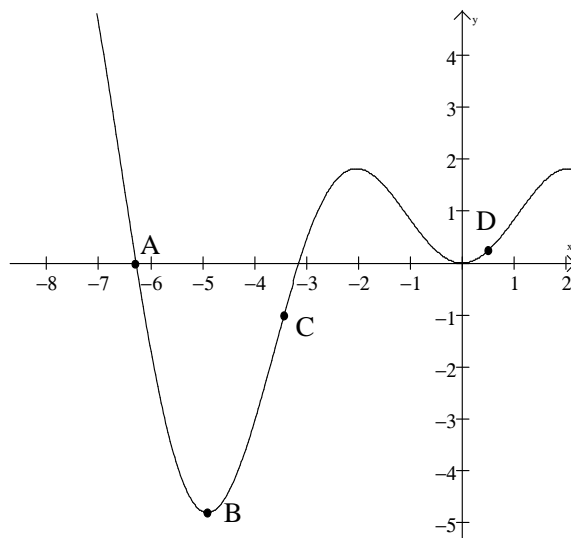


27. The graph of  $y + 1 = (x + 6)^2$  is given below. Which of the following graphs would result if this graph was reflected and translated 3 units to the right?



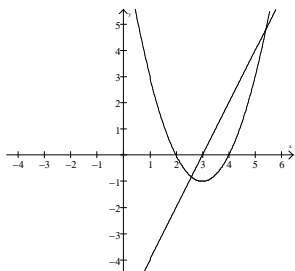
28. At which point is the instantaneous rate of change zero?

- (A) A
- (B) B
- (C) C
- (D) D

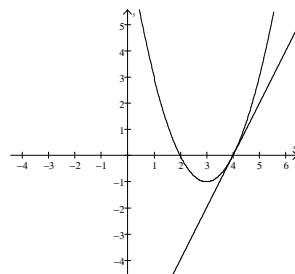


29. Which graph best shows a negative average rate of change?

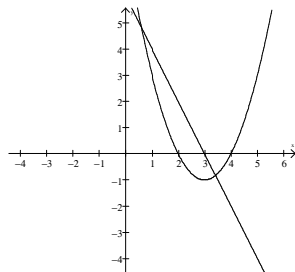
(A)



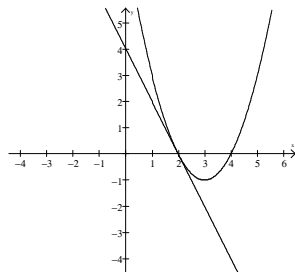
(B)



(C)

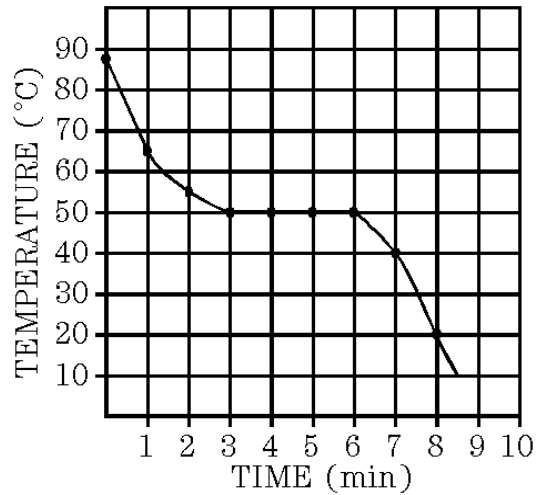


(D)



30. What is the average rate of change in temperature between 3 minutes and 8 minutes?

- (A) -6
- (B)  $-\frac{3}{5}$
- (C)  $-\frac{1}{6}$
- (D) 6



31. The tables below were generated from a continuous function. Which of the following best approximates the instantaneous rate of change at 2.0?

| $x$ | $y$   |
|-----|-------|
| 1.0 | 9.00  |
| 1.5 | 11.50 |
| 1.7 | 12.78 |
| 1.9 | 14.22 |

| $x$ | $y$   |
|-----|-------|
| 2.1 | 15.82 |
| 2.3 | 17.58 |
| 2.6 | 20.52 |
| 3.0 | 25.00 |

- (A) 7.6
- (B) 8.0
- (C) 8.6
- (D) 23

32. Which pattern represents a geometric sequence?

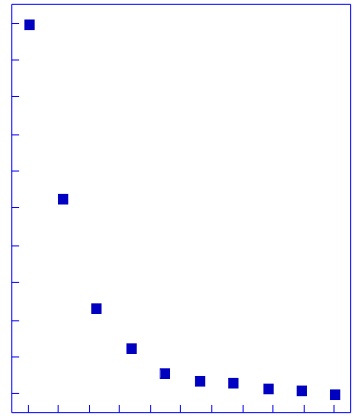
- (A)  $\{5, -1, -11, -25, -43, \dots\}$
- (B)  $\{5, 3, 1, -1, -5, \dots\}$
- (C)  $\{-2, -16, -54, -154, \dots\}$
- (D)  $\{-4, -8, -16, -32, -64, \dots\}$

33. Which exponential function is represented by the data in the table?

| $x$ | 0 | 1   | 2    | 3     |
|-----|---|-----|------|-------|
| $y$ | 5 | 2.5 | 1.25 | 0.625 |

- (A)  $y = \frac{1}{2}(5)^x$
- (B)  $y = 2(5)^x$
- (C)  $y = 5\left(\frac{1}{2}\right)^x$
- (D)  $y = 5(2)^x$

34. Which situation is best represented by the scatter plot?
- (A) The rise and fall of the tide over a period of time.  
 (B) Water poured from container at a constant rate.  
 (C) A car's value estimated over a period of time.  
 (D) Bacteria growth in a culture in the lab.



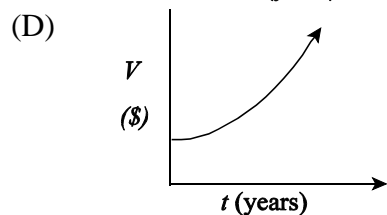
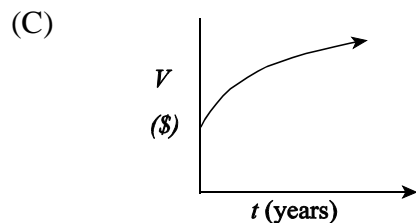
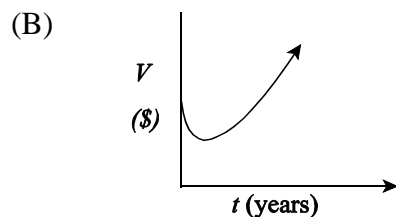
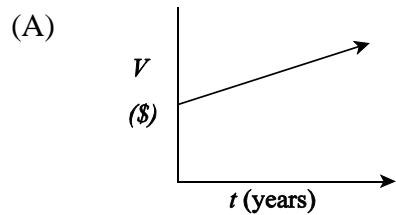
35. What is the value of  $(4^0 + 3^{-1})^{-2}$  ?

- (A)  $\frac{9}{169}$   
 (B)  $\frac{9}{16}$   
 (C)  $\frac{16}{9}$   
 (D)  $\frac{169}{9}$

36. The cooling of a cup of tea is modeled by  $T = 69(0.91)^{\frac{m}{8}} + 21$ , where  $T$  is the temperature in degrees Celsius and  $m$  is the time in minutes. What was the initial temperature of the tea?

- (A) 21  
 (B) 69  
 (C) 90  
 (D) 91

37. Which graph best models a deposit of \$500 with an interest rate of 5% compounded annually?

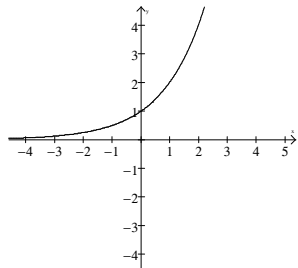


38. What is the range of  $y = 2(3)^x + 4$ ?

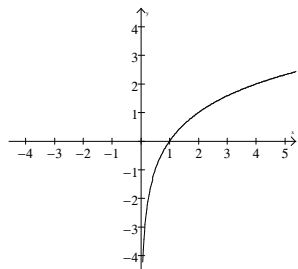
- (A)  $\{y | y \in \mathbf{R}\}$
- (B)  $\{y | y > 4; y \in \mathbf{R}\}$
- (C)  $\{y | y > 6; y \in \mathbf{R}\}$
- (D)  $\{y | y > 10; y \in \mathbf{R}\}$

39. Which graph represents a function of the form  $y = a \cdot b^x$ , where  $a \geq 1$  and  $b > 1$ ?

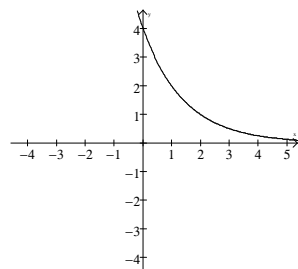
(A)



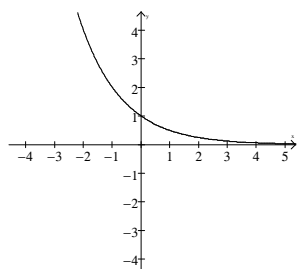
(B)



(C)



(D)

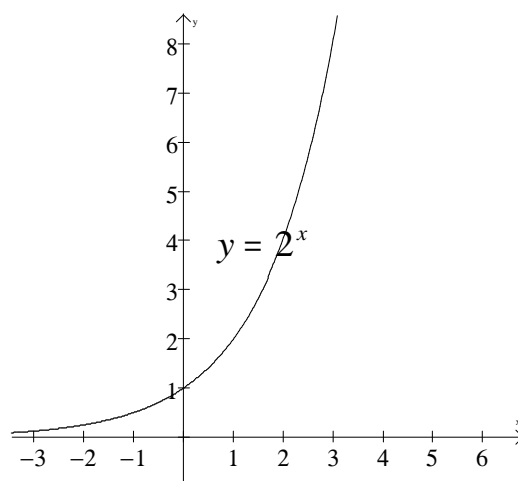


40. Which equation produces an exponential decay curve?

- (A)  $y = \frac{1}{2}(3)^x$
- (B)  $y = 3\left(\frac{1}{2}\right)^x$
- (C)  $y = \left(\frac{3}{2}\right)^x$
- (D)  $y = (1.2)^x$

41. What is the approximate value of  $x$ , if  $3 = 2^x$ ?

- (A) 0.5
- (B) 1.6
- (C) 3
- (D) 8



42. Solve for  $x$ :  $7^{\frac{x}{2}} = 49$

- (A) 1
- (B) 2
- (C) 4
- (D) 7

43. What is the  $y$ -intercept of the graph defined by the equation  $y = 5\left(\frac{1}{4}\right)^x - 1$ ?

- (A)  $(0, -1)$
- (B)  $(0, \frac{1}{4})$
- (C)  $(0, 4)$
- (D)  $(0, 5)$

44. Which function describes the data in the table?

|     |    |   |    |    |
|-----|----|---|----|----|
| $x$ | -2 | 0 | 2  | 4  |
| $y$ | 3  | 6 | 12 | 24 |

- (A)  $y = -1(3)^x$
- (B)  $y = 6\left(\frac{1}{2}\right)^{\frac{x}{2}}$
- (C)  $y = 3(2)^x$
- (D)  $y = 6(2)^{\frac{x}{2}}$

45. Find  $f(-1)$  for the function  $f(x) = 3^{-(2-x)}$ .

- (A) -27
- (B)  $\frac{-5}{3}$
- (C)  $\frac{1}{27}$
- (D)  $\frac{1}{3}$

46. If a piece of property valued at \$10 000 appreciates by 18% every two years, which function models the value  $v$  of the property in  $t$  years?

- (A)  $v = 10000(0.18)^{\frac{t}{2}}$
- (B)  $v = 10000(0.18)^{2t}$
- (C)  $v = 10000(1.18)^{\frac{t}{2}}$
- (D)  $v = 10000(1.18)^{2t}$

47. Which table describes a geometric sequence?

(A)

|     |    |   |               |                |                |
|-----|----|---|---------------|----------------|----------------|
| $x$ | 0  | 1 | 2             | 3              | 4              |
| $y$ | 20 | 5 | $\frac{5}{4}$ | $\frac{5}{16}$ | $\frac{5}{48}$ |

(B)

|     |   |   |    |    |    |
|-----|---|---|----|----|----|
| $x$ | 0 | 1 | 2  | 3  | 4  |
| $y$ | 3 | 9 | 21 | 30 | 40 |

(C)

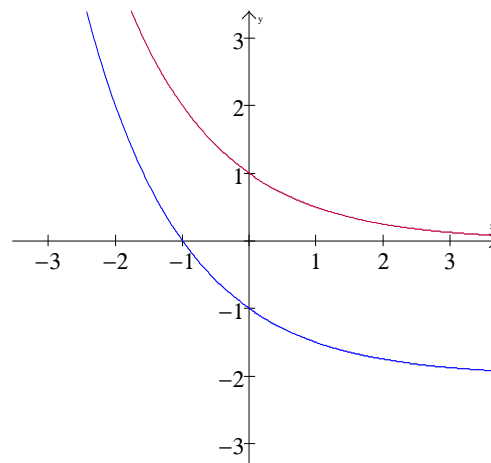
|     |   |   |   |   |    |
|-----|---|---|---|---|----|
| $x$ | 0 | 1 | 2 | 3 | 4  |
| $y$ | 2 | 4 | 6 | 8 | 10 |

(D)

|     |    |   |   |   |     |
|-----|----|---|---|---|-----|
| $x$ | 0  | 1 | 2 | 3 | 4   |
| $y$ | 20 | 5 | 2 | 1 | 0.5 |

48. The graphs  $y = \left(\frac{1}{2}\right)^x$  and  $y = \left(\frac{1}{2}\right)^x + k$  are shown. What is the value of  $k$ ?

- (A) -2
- (B) -1
- (C) 1
- (D) 2



49. Solve for  $x$ :  $\left(\frac{1}{3}\right)^{x-1} = (9)^{-5}$

- (A) 4
- (B) 6
- (C) 9
- (D) 11

50. When graphed, which equation has a negative  $y$ -intercept?

- (A)  $y - 4 = 2^x$
- (B)  $y = 2^x - 1$
- (C)  $y = 2^x + 1$
- (D)  $y + 3 = 2^x$

**PART II**  
**Total Value 50%**

Answer **ALL** items in the space provided. Show **ALL** workings.

*Value*

4 51. Algebraically determine the **EXACT** roots in simplest form for  $3(x^2 - 2x) = -11$

4 52. A parabola has vertex  $(2,8)$  and passes through the point  $(4,0)$ . Write the equation of the parabola in transformational form.

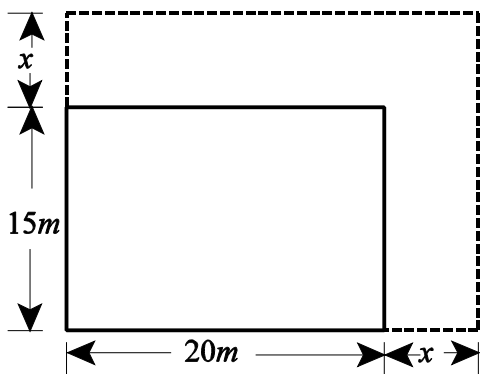


4 53. Change  $y = -2x^2 + 4x - 3$  to transformational form.

4 54. A person diving from a  $3m$  high diving board has his height above the water,  $h$ , in metres,  $t$  seconds after diving, given by  $h(t) = -5t^2 + 9t + 3$ . Algebraically determine the maximum height reached by the diver and the time taken to reach this height.

- 4 55. Two numbers differ by 18 and have a minimum product. Write a quadratic equation and use it to find the numbers.

- 4 56. An outdoor skating rink measures  $15m$  by  $20m$ . A strip of uniform width  $x$  is added along one end and one side, as in the diagram. As a result the new area is  $546m^2$ . Write a quadratic function which models the new rink's area and use it to find its dimensions.



4 57. A rocket is shot into the air from a platform. Its height above the ground, in metres,  $t$  seconds after it is launched, is approximated by  $h(t) = -5t^2 + 85t + 6$ . Algebraically determine the time it takes the rocket to hit the ground.

4 58. A person chewing gum blows a bubble. If the formula for the volume of the bubble is  $V = \frac{4}{3}\pi r^3$ . Calculate the average rate of change when the radius increases from  $1\text{ cm}$  to  $3\text{ cm}$ .

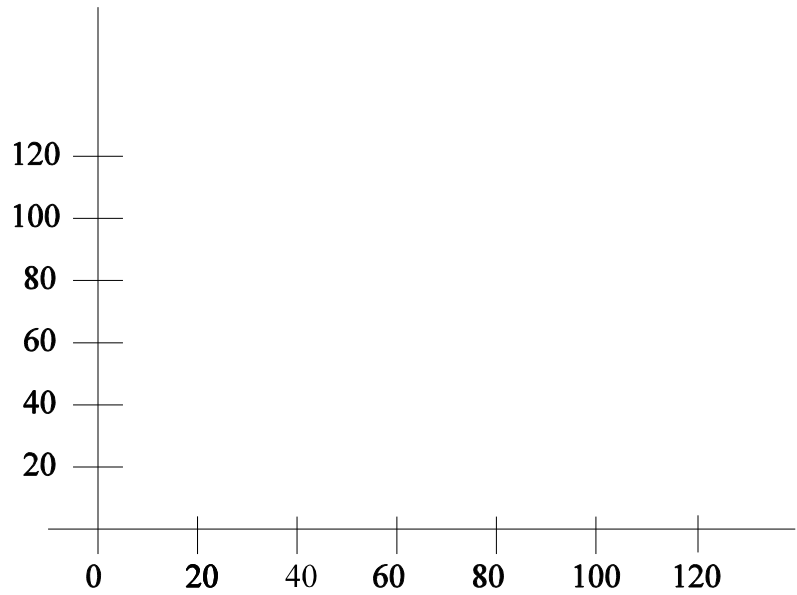
4 59. A soccer player kicks a soccer ball into the air. The ball's height,  $h$ , in metres,  $t$  seconds after it is kicked is given by  $h(t) = 1 + 28t - 4.9t^2$ . What is the approximate instantaneous rate of change in the height of the soccer ball at  $t = 3$  seconds?

3 60. Solve for  $x$ :  $4^{x-1} + 5 = 37$

4 61. A population of rabbits is known to double every three years. In 12 years the population grew to a total of 1920 rabbits. What was the initial number of rabbits?

62. The cooling of a cup of herbal tea is modeled by  $T = 77(0.91)^{\frac{t}{4}} + 21$ , where  $T$  is the temperature in degrees Celsius and  $t$  is the time in minutes. Complete the table of values and sketch the graph, including the asymptote, for the first 2 hours.

| $t$ | $T$ |
|-----|-----|
| 0   |     |
| 20  |     |
| 40  |     |
| 60  |     |
| 80  |     |
| 100 |     |
| 120 |     |



63. A geologist unearths a sample of a radioactive substance which has a half-life of 1.25 years. In 2006, there are 150g of the substance. Write an exponential function that models the situation and use it to algebraically determine how much will remain in 2014.