PART I

1	B	11	C	21	A	31	B	41	B
2	C	12	B	22	B	32	D	42	C
3	C	13	B	23	B	33	C	43	C
4	B	14	C	24	A	34	C	44	D
5	A	15	D	25	D	35	B	45	C
6	C	16	A	26	C	36	C	46	C
7	D	17	C	27	D	37	D	47	A
8	D	18	D	28	B	38	B	48	A
9	D	19	A	29	C	39	A	49	D
10	C	20	C	30	A	40	B	50	D

PART II

Value

4 51. Algebraically determine the **EXACT** roots in simplest form for $3(x^2 - 2x) = -11$

$3x^2 - 6x + 11 = 0 \quad \{1\}$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(11)}}{6} \quad \{0.5\}$

$x = \frac{6 \pm \sqrt{36 - 132}}{6} \quad \{0.5\}$

$x = \frac{6 \pm \sqrt{-96}}{6} \quad \{0.5\}$

$\frac{x = 6 \pm 4i\sqrt{6}}{6} \quad \{1\}$

$x = \frac{3 \pm 2i\sqrt{6}}{3} \quad \{0.5\}$

4 52. A parabola has vertex $(2,8)$ and passes through the point $(4,0)$. Write the equation of the parabola in transformational form.

$\frac{1}{a}(y - k) = (x - h)^2$

$\frac{1}{a}(y - 8) = (x - 2)^2 \quad \{1\}$

$\frac{1}{a}(0 - 8) = (4 - 2)^2 \quad \{1\}$

$\frac{1}{a}(-8) = (2)^2$

$\frac{1}{a}(-8) = 4 \quad \{0.5\}$

$\frac{1}{a} = -\frac{4}{8}$

$\frac{1}{a} = -\frac{1}{2} \quad \{1\}$

$\therefore$ the equation is: $-\frac{1}{2}(y - 8) = (x - 2)^2 \quad \{0.5\}$

4 53. Change $y = -2x^2 + 4x - 3$ to transformational form.

EITHER:

$y + 3 = -2(x^2 - 2x)$ {1}

$y + 3 - 2 = -2(x^2 - 2x + 1)$ {1}

$y + 1 = -2(x - 1)^2$ {1}

$-\frac{1}{2}(y + 1) = (x - 1)^2$ {1}

OR:

$x = \frac{-b}{2a} = \frac{-4}{2(-2)} = 1$ {1}

$y = -2(1)^2 + 4(1) - 3 = -1$ {1}

vertex $(1, -1)$

$a = -2$

$\therefore$ equation $-\frac{1}{2}(y + 1) = (x - 1)^2$ {2}

4 54. A person diving from a 3*m* high diving board has his height above the water, *h*, in metres, *t* seconds after diving, given by $h(t) = -5t^2 + 9t + 3$. Algebraically determine the maximum height reached by the diver and the time taken to reach this height.

$t = \frac{-b}{2a} = \frac{-9}{2(-5)} = \frac{-9}{-10} = 0.9s$ {2}

$h(0.9) = -5(0.9)^2 + 9(0.9) + 3 = 7.05m$ {2}

The maximum height of 7.05*m* was reached after 0.9 seconds.

4 55. Two numbers differ by 18 and have a minimum product. Write a quadratic equation and use it to find the numbers.

Let the numbers be *x* and *y* and the product be *P*.

$P = xy$

$x - y = 18$

$y = x + 18$ {1}

$P = x(x + 18)$

$P = x^2 + 18x$ {1}

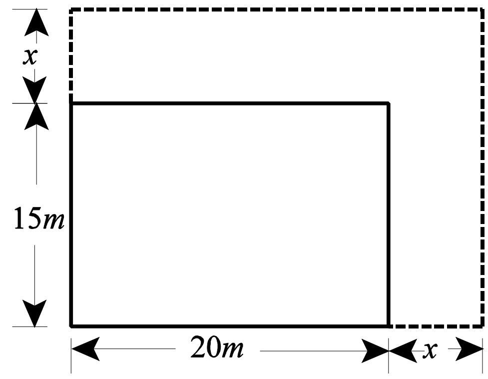
$x = \frac{-b}{2a}$

$x = \frac{-18}{2(1)}$

$x = -9$ {1}

$y = -9 + 18 = 9$ {1} $\therefore$ the numbers are -9 and 9.

4 56. An outdoor skating rink measures 15*m* by 20*m*. A strip of uniform width *x* is added along one end and one side, as in the diagram. As a result the new area is $546m^2$. Write a quadratic function which models the new rink's area and use it to find its dimensions.



$a = l \cdot w$

$(x+15)(x+20) = 546$ {1}

$x^2 + 35x + 300 = 546$ {0.5}

$x^2 + 35x - 246 = 0$ {0.5}

$(x+41)(x-6) = 0$ {0.5}

$x + 41 = 0$ OR $x - 6 = 0$

$x = -41$ OR $x = 6$ {0.5}

reject ↑

new width $= 6 + 15 = 21$ and new length $= 6 + 20 = 26$ {1}

The dimensions of the new rink are $21m \times 26m$.

4 57. A rocket is shot into the air from a platform. Its height above the ground, in metres, *t* seconds after it is launched, is approximated by $h(t) = -5t^2 + 85t + 6$. Algebraically determine the time it takes the rocket to hit the ground.

When it reaches the ground $h(t) = 0$

$\therefore \quad 0 = -5t^2 + 85t + 7$ {0.5}

$$t = \frac{-85 \pm \sqrt{7225 + 120}}{-10}$$ {1}

$$t = \frac{-85 \pm \sqrt{7345}}{-10}$$ {0.5}

$$t = \frac{-85 \pm 85.7}{-10}$$ {1}

$t = -0.07$ OR 17.1 {1}

reject ↑

The rocket hit the ground after 17.1 seconds.

4 58. A person chewing gum blows a bubble. If the formula for the volume of the bubble is $V = \frac{4}{3}\pi r^3$. Calculate the average rate of change when the radius increases from 1*cm* to 3*cm*.

$V(3) = \frac{4}{3}(3.14)(3)^3 = 113.1$ {1}

$V(3) = \frac{4}{3}(3.14)(1)^3 = 4.188$ {1}

$$A.R.O.C. = \frac{V(r_2) - V(r_1)}{r_2 - r_1}$$

$$A.R.O.C. = \frac{V(3) - V(1)}{3 - 1}$$ {1}

$$A.R.O.C. = \frac{113.1 - 4.188}{3 - 1}$$

$$A.R.O.C. = \frac{108.2}{2} = 54.1$$ {1}

The average rate of change is: $54.1 \, {}^{cm^3}\!/_{cm}$.

4 59. A soccer player kicks a soccer ball into the air. The ball's height, *h*, in metres, *t* seconds after it is kicked is given by $h(t) = 1 + 28t - 4.9t^2$. What is the approximate instantaneous rate of change in the height of the soccer ball at $t = 3$ seconds?

$h(3.01) = 1 + 28(3.01) - 4.9(3.01)^2 = 40.89m$ {1}

$h(2.99) = 1 + 28(2.99) - 4.9(2.99)^2 = 40.91m$ {1}

$$I.R.O.C. = \frac{h(t_2) - h(t_1)}{t_2 - t_1}$$

$$I.R.O.C. = \frac{h(3.01) - h(2.99)}{3.01 - 2.99}$$ {1}

$$I.R.O.C. = \frac{40.89 - 40.91}{3.01 - 2.99}$$

$$I.R.O.C. = \frac{-0.02}{0.02}$$

$$I.R.O.C. = -1.4$$ {1}

The approximate instantaneous rate of change is: $-1.4 \, {}^{m}\!/_{s}$.

NOTE: $h(3.1)$ and $h(2.9)$ is also acceptable.

3 60. Solve for x: $4^{x-1} + 5 = 37$

$4^{x-1} = 37 - 5$

$4^{x-1} = 32$ $\{0.5\}$

$(2^2)^{x-1} = 2^5$ $\{0.5\}$

$2^{2x-2} = 2^5$ $\{0.5\}$

$2x - 2 = 5$ $\{0.5\}$

$2x = 5 + 2$

$2x = 7$ $\{0.5\}$

$x = \frac{7}{2}$ $\{0.5\}$

4 61. A population of rabbits is known to double every three years. In 12 years the population grew to a total of 1920 rabbits. What was the initial number of rabbits?

N = population of rabbits after ***t*** years = 1920
t = time in years=12
a = initial number of rabbits = ?
b = base (doubling) = 2
c = doubling time = 3

$N = a \cdot b^{\frac{t}{c}}$

$1920 = a(2)^{\frac{12}{3}}$ $\{2\}$

$1920 = a(2)^4$ $\{0.5\}$

$1920 = 16a$ $\{0.5\}$

$1920/16 = a$ $\{0.5\}$

$120 = a$ $\{0.5\}$

The initial number of rabbits was 120

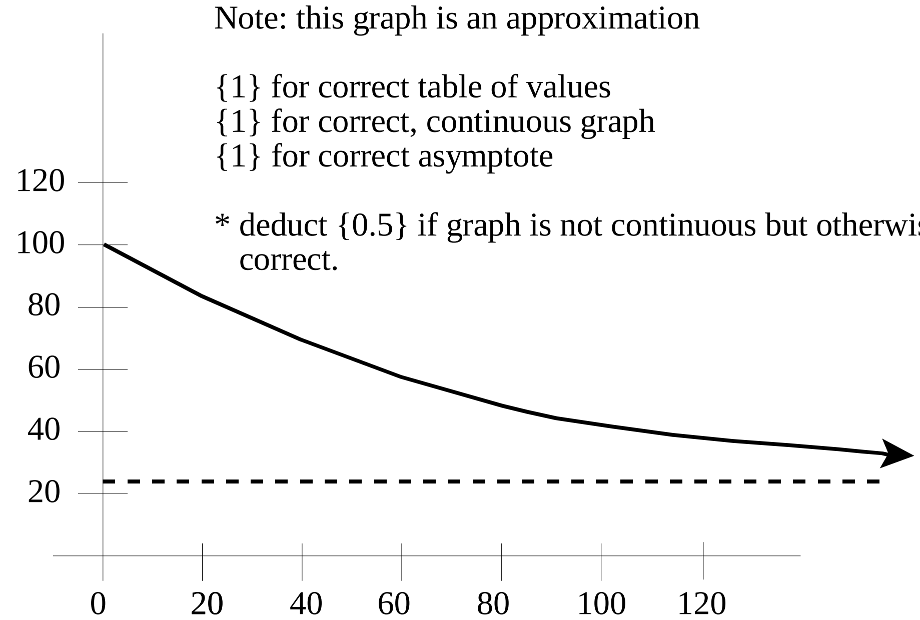
3 62. The cooling of a cup of herbal tea is modeled by the equation $T = 77(0.91)^{\frac{t}{4}} + 21$ where T is the temperature in degrees Celsius and t is the time in minutes. Complete the table of values and sketch the graph, including the asymptote, for the first 2 hours.

t	***T***
0	98
20	66
40	47.8
60	36.9
80	30.4
100	26.5
120	24

Note: this graph is an approximation

{1} for correct table of values
{1} for correct, continuous graph
{1} for correct asymptote

* deduct {0.5} if graph is not continuous but otherwise correct.



4 63. A geologist unearths a sample of a radioactive substance which has a half-life of 1.25 years. In 2006 there are 150g of the substance. Write an exponential function that models the situation and use it to algebraically determine how much will remain in 2014.

A = amount remaining in years = ?
t = time in years (2014 - 2006) = 8 {1}
a = initial amount =150*g*
b = base (half-life) = ½
c = half-life of substance =1.25

$$A = a \cdot b^{\frac{t}{c}}$$

$$A = 150\left(\tfrac{1}{2}\right)^{\frac{8}{1.25}} \qquad \{2\}$$

$$A = 150\left(\tfrac{1}{2}\right)^{6.4}$$

$$A = 150(0.0118)$$

$$A = 1.7g \qquad \{1\}$$

The amount remaining is 1.7 grams.