COMMON EXAMINATION

MATHEMATICS 2204

JUNE 2009

Answer Sheet Section A

1.	A
2.	B
3.	A
4.	C
5.	A
6.	A
7.	D
8.	B
9.	C
10.	B
11.	B
12.	B
13.	A
14.	B
15.	D
16.	C
17.	C
18.	C
19.	C
20.	A
21.	C
22.	C
23.	D
24.	A
25.	C

26.	A
27.	C
28.	D
29.	A
30.	D
31.	C
32.	A
33.	A
34.	B
35.	C
36.	D
37.	A
38.	A
39.	D
40.	C
41.	C
42.	D
43.	C
44.	B
45.	B
46.	B
47.	A
48.	A
49.	D
50.	D

SECTION B
Total Value: 50%

Answer ALL items in the space provided. Show ALL workings.

4 51. Solve the following system of equations using either substitution or elimination:

Equation 1: $x + y + 2z = 0$
Equation 2: $3x + 2y - z = 13$
Eqaution 3: $x - 2y + z = -1$

By substitution:

1 pt Solve Equation 2 for *z*: $z = 3x + 2y - 13$.

Substitute the new equation into Equation 1 and Equation 3:

1 pt $x + y + 2(3x + 2y - 13) = 0 \rightarrow 7x + 5y = 26$

1 pt $x - 2y + (3x + 2y - 13) = -1 \rightarrow 4x + 0y = 12$

The last equation gives $x = 3$. This gives $y = 1$ and $z = -2$. Therefore, the solution is

1 pt $\boxed{(3,1,-2)}$

By elimination:

Add Equation 2 and Equation 3:

$$\begin{array}{ccccccc} 3x & + & 2y & - & z & = & 13 \\ x & - & 2y & + & z & = & -1 \\ \hline 4x & & & & & = & 12 \end{array}$$ **1 pt**

This gives $x = 3$. Next multiply Equation 1 by 2 and add it to Equation 3: **1 pt**

$$\begin{array}{ccccccc} 2x & + & 2y & + & 4z & = & 0 \\ x & - & 2y & + & z & = & -1 \\ \hline 3x & & & + & 5z & = & -1 \end{array}$$ **1 pt**

Since $x = 3$, we get $z = -2$ from the above equation. Finally, we use Equation 1 to get $y = 1$. Therefore, the solution is $\boxed{(3,1,-2)}$ **1 pt**

4 52. Barry's Bakery sells cookies and muffins. One customer orders 5 muffins and 8 cookies, and is charged $17.50. Another customer orders 3 muffins and 5 cookies, and is charged $10.75. Create a system of equations to represent this information and, **<u>using matrices</u>**, find the price of each item.

Let *c* be the price of cookies and *m* be the price of cookies. Then:

$$5m + 8c = 17.50$$
$$3m + 5c = 10.75$$ **1 pt**

Writing as a matrix:

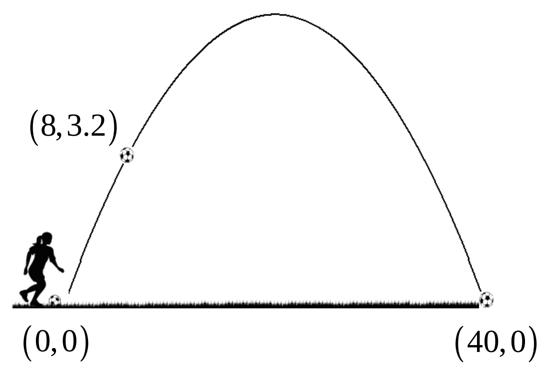
$$\begin{bmatrix} 5 & 8 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} 17.50 \\ 10.75 \end{bmatrix}$$ **1 pt**

Students can solve using the matrix function on their calculators, or algebraically using the inverse matrix method.

Solving, we get $\boxed{m = \$1.50}$ and $\boxed{c = \$1.25}$
1 pt **1 pt**

4 53. Amy kicked a soccer ball. After the ball travelled a distance of 8 metres, it was at a height of 3.2 metres. When the ball hit the ground, it was 40 metres away from Amy. Set up and solve a system of equations to determine the quadratic function that models the path of the soccer ball.

Let x be the ground distance in metres and y be the height of the soccer ball in metres.



Therefore, $y = ax^2 + bx + c$. Since the parabola passes though $(0,0)$, we have $c = 0$. Therefore, $y = ax^2 + bx$. **0.5 pt**

Next, we substitute the remaining two points into the above expression:

$(8,3.2)$	$(40,0)$
$3.2 = a(8)^2 + b(4) \rightarrow 64a + 8b = 3.2$	$0 = a(40)^2 + b(40) \rightarrow 1600a + 40b = 0$
1 pt	**1 pt**

Finally, we solve the system $\begin{matrix} 64a + 8b = 3.2 \\ 1600a + 40b = 0 \end{matrix}$ by any method to get:

$$a = -0.0125 \qquad b = 0.5$$ **1 pt**

Therefore, our function is $\boxed{y = -0.0125x^2 + 0.5x}$, which can also be written as $\boxed{y = -\tfrac{1}{80}x^2 + \tfrac{1}{2}x}$ **0.5 pt**

3 54. Graph the following relation: $-\tfrac{1}{2}(y+1) = \sin 3(x - 90°)$

Using the mapping rule:

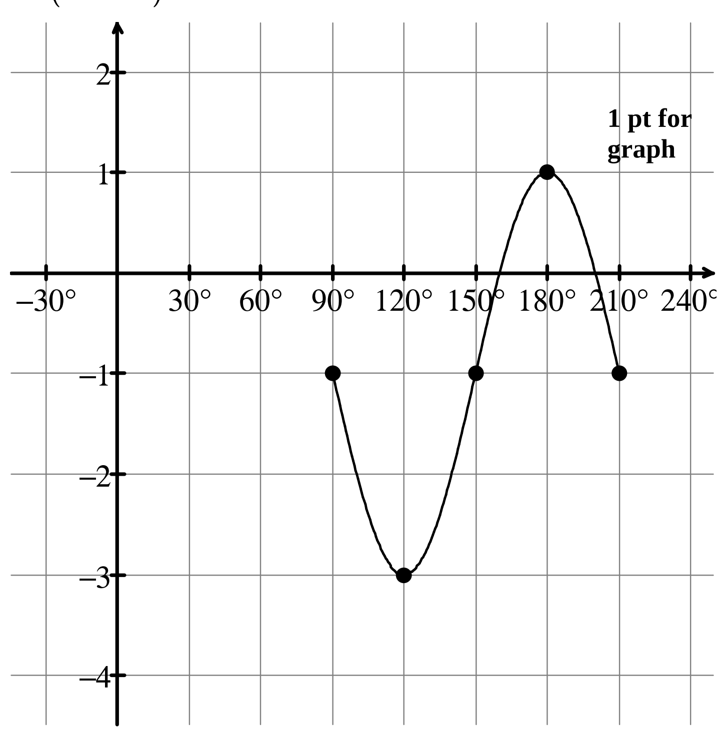
The mapping rule is:

$(x, y) \rightarrow \left(\tfrac{1}{3}x + 90°, -2y - 1\right)$ **1 pt**

This gives a table of values of:

x	y
90°	−1
120°	−3
150°	−1
180°	1
210°	−1

1 pt



Using transformations:

Reflection: yes

HT: 90°

VT: −1 (giving a sinusoidal axis of $y = -1$)

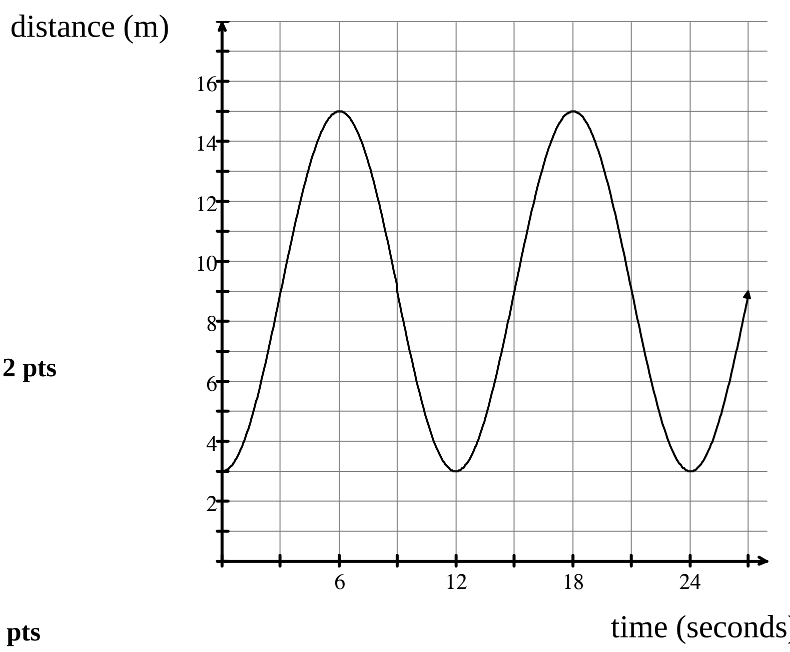
HS: $\tfrac{1}{3}$ (giving a period of 120°)

VS: 2 (giving an amplitude of 2)

2 pts

4 55. A mother puts her child on a Merry-Go-Round. To watch her child, she stands at a point that is initially 3 metres away from the child, which is the closest distance between the mother and child. At 6 seconds, the child is 15 metres from his mother, which is the farthest distance between the mother and child. Assuming the distance between the mother and child varies sinusoidally with time, determine the relation that models this situation.

Note: There are an infinite number of answers, depending on the initial point and on the choice of functions. For this answer, we will write the graph as an image of $y=\cos(x)$ and start at $(0,0)$:



Reflection:	Yes
HT:	0
VT:	9
HS:	$\frac{\text{Period}}{360}=\frac{12}{360}=\frac{1}{30}$
VS:	6

2 pts

Therefore, the mapping rule is $(x,y)\to\left(\frac{1}{30}x,-6y+9\right)$ and the equation is $\boxed{-\frac{1}{6}(y-9)=\cos(30x)}$ **2 pts**

3 56. Simplify the following expression:

$$\frac{1}{x+2}+\frac{2}{x^2+2x}$$

$$\frac{1}{x+2}+\frac{2}{x^2+2x}=\frac{1}{x+2}+\frac{2}{x(x+2)}$$ **1 pt**

$$=\frac{1}{x+2}\times\frac{x}{x}+\frac{2}{x(x+2)}$$

$$=\frac{x}{x(x+2)}+\frac{2}{x(x+2)}$$ **1 pt**

$$=\frac{x+2}{x(x+2)}$$

$$\boxed{=\frac{1}{x}}$$ **1 pt**

3 57. Prove: $\sec\theta\left(1-\sin^2\theta\right)=\cos\theta$.

$$
\begin{aligned}
LHS &= \sec\theta\left(1-\sin^2\theta\right) \\
&= \frac{1}{\cos\theta}\left(1-\sin^2\theta\right) && \textbf{1 pt} \\
&= \frac{1}{\cos\theta}\left(\cos^2\theta\right) && \textbf{1 pt} \\
&= \cos\theta && \textbf{1 pt} \\
&= RHS
\end{aligned}
$$

4 58. Solve $\left(2\sin\theta-1\right)\left(\sin\theta-1\right)=0$ where $0°\le\theta<360°$.

We need to solve two equations:

$$
\begin{aligned}
2\sin\theta-1&=0 \\
\sin\theta&=\tfrac{1}{2} \\
\theta&=30°,150°
\end{aligned}
$$

2 pts

and

$$
\begin{aligned}
\sin\theta-1&=0 \\
\sin\theta&=1 \\
\theta&=90°
\end{aligned}
$$

2 pts

Therefore, $\boxed{\theta=30°,90°,\text{ and }150°}$

3 59. Simplify the following expression, expressing your answer in EXACT simplest form:

$$\sin 120° \cos 45° - \cos 240° \sin 90°$$

$$\sin 120° \cos 45° - \cos 240° \sin 90° = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)(1)$$ **1 pt**

$$= \frac{\sqrt{6}}{4} + \frac{1}{2}$$ **1 pt**

$$\boxed{= \frac{\sqrt{6}+2}{4}}$$ **1 pt**

3 60. Simplify the following expression, **listing all restrictions**:

$$\frac{2y+6}{y^2-9} \times \frac{y^2+2y-15}{3y+15}$$

$$\frac{2y+6}{y^2-9} \times \frac{y^2+2y-15}{3y+15} = \frac{2(y+3)}{(y-3)(y+3)} \times \frac{(y+5)(y-3)}{3(y+5)}$$

Restrictions: $x \neq \pm 3, -5$

$$= \frac{2\cancel{(y+3)}}{\cancel{(y-3)}\cancel{(y+3)}} \times \frac{\cancel{(y+5)}\cancel{(y-3)}}{3\cancel{(y+5)}}$$

$$\boxed{= \frac{2}{3}}$$

Give 1 point for factoring correctly, 1 point for the correct answer, and 1 point for the restrictions.

61. Good Day Tires claims that their new line of tires will last for 80000 km. A consumer research group decides to test this claim. The group randomly selects 400 tires and tests each tire. The data from this sample shows that the mean life of a tire is 78000 km, with a standard deviation of 1500 km.

3 (A) Algebraically determine the 95% confidence interval for the mean life of a tire.

The margin of error is given by $MOE = 1.96\frac{S_x}{\sqrt{n}} = 1.96\left(\frac{1500}{\sqrt{400}}\right) = 147$ **1 pt**

Therefore, the 95% confidence interval is:

$$(78000-147, 78000+147) = \boxed{(77853, 78146)}$$ **2 pts**

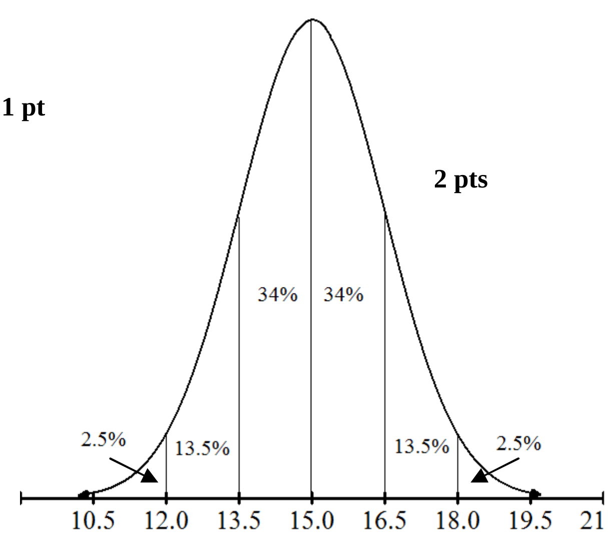
1 (B) Does the confidence interval support the claim of Good Day Tires? Explain your answers.

Since the company claims that the tire will last for 80000 km and this number falls outside of the 95% confidence interval, this data **does not** support the claim of Good Day Tires. **1 pt**

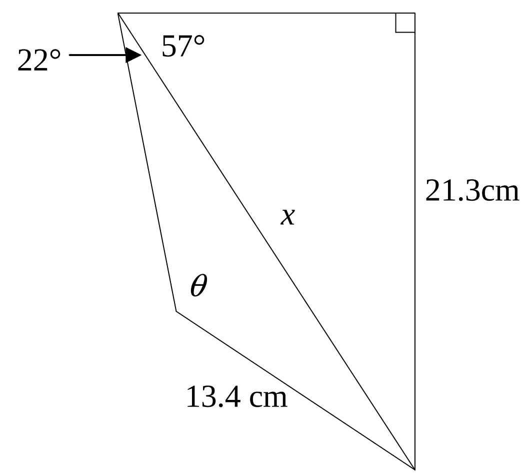
3 62. The number of chocolate chips in a certain brand of chocolate chip cookies is known to be normally distributed with a mean of 15 chips per cookie and a standard deviation of 1.5. Draw and label a normal distribution curve for this situation, and determine the percentage of cookies has between 12 and 16.5 chocolate chips.

We would expect $13.5 + 34 + 34 = 81.5\%$ of the chocolate chip cookies to have between 12 and 16.5 chocolate chips. **1 pt**

2 pts



4 63. Algebraically determine the measure of θ given that θ is obtuse.



Use right angle trigonometry to find *x*:

$\sin 57° = \frac{21.3}{x} \rightarrow x = 25.4\text{cm}$ **1 pt**

Using the Law of Sines:

$$\frac{\sin\theta}{25.4} = \frac{\sin 22°}{13.4}$$
$$\sin\theta = \frac{25.4\sin 22°}{13.4}$$
$$\sin\theta = 0.7101$$

This gives $\theta = 45°$ **2 pts**

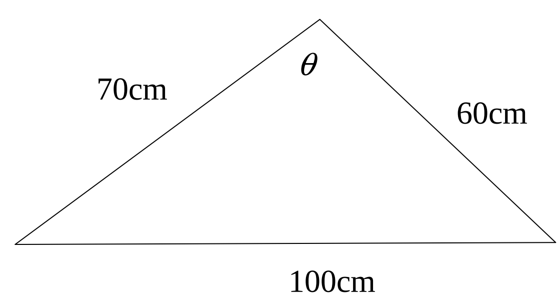
or $\theta = 180° - 45° = 135°$

Since θ is obtuse, we have $\boxed{\theta = 135°}$ **1 pt**

64. A triangular sign needs to be painted. The sides measure 60cm, 70cm, and 100cm.

3 (A) Determine value of θ and the area of the triangle.

Students must first use the Law of Cosines to determine an angle:



$$100^2 = 70^2 + 60^2 - 2(70)(60)\cos\theta$$
$$10000 = 4900 + 3600 - 8400\cos\theta$$
$$1500 = -8400\cos\theta$$
$$\cos\theta = -0.1786$$
$$\theta = 100°$$ **2 pt**

Therefore, the area is $A = \frac{1}{2}(70)(60)\sin(100°) = \boxed{2068.1\text{cm}^2}$ **1 pt**

1 (B) If a can of paint costs \$7.50 can covers 520cm², determine the cost to paint the sign.

It requires $\frac{2068.1}{520} = 4.0$ cans of paint to cover the sign. Therefore, it will cost $\$7.50 \times 4 = \boxed{\$30}$. **1 pt**